# STATISTICAL FAILURE ANALYSIS OF HIGH VOLTAGE SF6 CIRCUIT BREAKERS

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**Abstract**: Simple functional forms are assumed for the statistical models, and therefore they are usually Weibull and Normal (or Lognormal) distributions for high voltage $SF_6$ circuit – breakers. The kind type of model is based on the empirical data obtained from the statistical surveys on failure, and may use goodness – of – fit test for the choice of failure – time distribution. The parameters of the distribution are based on the reliability data which are determined by the least-squares technique or the maximum likelihood method. In this paper, a statistical approach is developed for individual characteristic parameter of the high voltage circuit breakers such as $SF_6$ leakage, $N_2$ leakage, or oil leakage and control- and protection problem, etc.

## 1 INTRODUCTION

In general, fitting a theoretical distribution is preferred over empirically developing a model. First, empirical models do not provide information beyond the range of the sample data. In reliability engineering the tails of the distribution are of most interest. Second, there is an interest in determining the probabilistic nature of the underlying failure process. A sample is only a small random part of the population of failure times, and it is the distribution the sample came from and not the sample itself that we want to establish. Third, often the failure process is a result of some physical phenomena that can be associated with a particular distribution. Fourth, small sample sizes provide very little information concerning the failure process. However, if the sample is consistent with a theoretical distribution, then much stronger results based on the properties of the theoretical distribution are possible. Fifth, the traditional parameters of a statistical model (e.g., mean time to failure and standard deviation) are not of primary interest. Instead, reliability research is interested in specific reliability measures or particular characteristics of a failure-time distribution (e.g., failure probabilities, failure rates, remaining lifetime, etc.). Finally, use can be made of the theoretical reliability model in performing more complex analysis of the failure process. Without an analytical reliability model to use, it is difficult to derive more complicated relationships, such as the preventive maintenance reliability model.

The primary approach taken so far is to treat the occurrence of failures as a random process. As a consequence, a reliability model must be developed statistically. Through goodness-of-fit tests the analysis of failure data can be performed in deriving an acceptable reliability model. The methods commonly used for parameter estimation are maximum likelihood estimator and best linear unbiased estimator. Based on massive measured data, it is reasonable to ask which statistical distribution or model parameter is better and more appropriate in terms of such failure criteria.

## 2 FAILURE PROBABILITY

It is commonly recognized that the statistical characteristics of component can be represented by appropriate distribution functions. For the statistical evaluation of data obtained from the 550 kV $SF_6$ circuit breakers, the most popular models are Weibull distribution and lognormal distribution. The failure probability $F(t)$ of the Weibull distribution is given by

$$F(t)=1-\exp\left[-\left(\frac{t}{\theta}\right)^{\gamma}\right] \tag{1}$$

where $\gamma$ and $\theta$ are the shape and scale parameters, respectively.

A typical expression for the failure rate $h(t)$ is

$$h(t)=\frac{\gamma}{\theta}\cdot\left(\frac{t}{\theta}\right)^{\gamma-1} \tag{2}$$

Let $t$ be the natural or logarithm of the failure time for normal or lognormal distribution respectively. The cumulative distribution function of normal or lognormal distribution is

$$F(t)=\Phi\left(\frac{t-\mu}{\sigma}\right) \tag{3}$$

where $\mu$, $\sigma$ and $\Phi$ are the mean value, the standard deviation and the cumulative normal distribution function.

The cumulative normal distribution function $\Phi$ is defined as

$$\Phi(t)=\int_0^t \frac{1}{\sqrt{2\pi x}}\cdot\exp\left(-\frac{x^2}{2}\right)dx \tag{4}$$

Thus, the failure rate function of the normal distribution is

$$h(t)=\frac{\Phi\left(\frac{t-\mu}{\sigma}\right)}{\sigma\left[1-\Phi\left(\frac{t-\mu}{\sigma}\right)\right]} \tag{5}$$

The reliability function of the normal distribution *R*(*t*) is given below

When multiply censored data are present, the likelihood function must be modified to reflect the fact that at the censored times no failure occurred. The objective of the method is to derive, directly from the failure times, the failure distribution. Let $t_i$ be the ordered failure time and $n_i$ be the size at risk, a logical estimate for the distribution function *F*(*t*) is

$$F(t)=1-\prod_{i=1}^{n}\left(1-\frac{1}{n_i}\right) \tag{6}$$

For the collected data, one should hypothesize a distribution that the failure times came from the specified distribution. It is for these reasons that a goodness-of-fit test is applied for each of the distributions. The goodness of fit of a statistical model describes how well it fits a set of observations. Measures of goodness of fit typically summarize the discrepancy between observed values and the expected values under the model in question. One way is to construct a sum of squared errors divided by the variance of the measurement error $\sigma^2$:

$$\chi^2=\sum\frac{(O-E)^2}{\sigma^2} \tag{7}$$

where *O* and *E* are the observed and expected number of failures respectively.

Once one or more distributions have been identified, we need to estimate the parameters of the probability distribution that describes the failure time of the population subjected to the observation. Clearly, the accuracy of the estimate of the parameters depends on the sample size and the method used for estimating the parameters. The maximum likelihood estimator (MLE) is the preferred estimates of the distribution parameters. This can be accomplished by defining the likelihood function in the following manner:

$$L(\zeta)=\prod_F f(t;\zeta)\prod_C R(t^+;\zeta) \tag{8}$$

where $\zeta$ is the unknown parameter, *F* is the set of indices for the failure times, and *C* is the set of indices for the censored times. $t^+$ is a censored time.

We take the logarithm of the equation (8) and then the derivatives of the logarithmic function with respect to $\gamma$ and $\theta$. This results in the following two equations:

$$\frac{1}{\gamma}+\sum_{i\in F}\frac{\ln t_i}{r}-\sum t_i^\gamma\cdot\ln t_i\sum\left(t_i^\gamma\right)^{-1}=0 \tag{9}$$

$$-\frac{1}{\theta^\gamma}+\sum\frac{t_i^\gamma}{r}=0 \tag{10}$$

where *r* is the number of censored components. The MLE of $\gamma$ and $\theta$ can be obtained by solving the equations (9) and (10) simultaneously.

Let $t_i$ be the natural or logarithm of the *i*-th failure or censored time for normal or lognormal distribution. Define the failure rate

$$h(z_i)=\frac{\phi(z_i)}{\bar{\Phi}(z_i)} \tag{11}$$

$$z_i=\frac{t_i-\mu}{\sigma} \tag{12}$$

Given initial values for $\mu$ and $\sigma$, recursively solve for $\mu$ and $\sigma$

$$\sum_{i\in F}z_i+\sum_{i\in C}h(z_i)=0 \tag{13}$$

$$-r+\sum_{i\in F}z_i^2+\sum_{i\in C}z_i h(z_i)=0 \tag{14}$$

The remaining lifetime *RLT*(*t*) can be determined by the failure probability *F*(*t*):

$$RLT(t)=\int_t^\infty\frac{1-F(x)}{1-F(t)}dx \tag{15}$$

## 3 CASE STUDIES

To simulate the failure probability and to estimate the failure rate and the remaining lifetime of the high voltage $SF_6$ circuit breakers, the statistical survey on failure probability with the operational history was gathered from the practical operational experiences. The useful information about the occurrences of failure with the location and the mode of failures for 550 kV $SF_6$ circuit breakers is given by [1-2]. The set of data comes from a large population of 550 kV $SF_6$ circuit breakers with a relatively large number of failures. These data help us to develop the reliability model.

We will test via simulation which theoretical model will provide the most accurate estimates. A goodness-of-fit test defines the best fit as one that minimizes the sum of squared errors between the observed data and the fitted distribution (Table 1). As a rule of thumb, a large $\chi^2$ indicates a poor model fit. However $\chi^2$<1 indicates that the model is over-fitting the data. In principle a value of $\chi^2$=1 indicates that the extent of the match between observations and estimates is in accord with the error variance.

**Table 1:** Distributions with the model parameter according to the goodness-of-fit test

| No | $\gamma$ | $\theta$ | $\chi^2$ | $\mu$ | $\sigma$ | $\chi^2$ |
|---|---|---|---|---|---|---|
| 1 | 1.55 | $1.85\cdot10^4$ | 1.26 | $1.05\cdot10^4$ | $4.28\cdot10^3$ | 1.22 |
| 2 | 4.35 | $8.85\cdot10^3$ | 1.09 | $8.16\cdot10^3$ | $2.25\cdot10^3$ | 1.10 |
| 3 | 0.78 | $3.9\cdot10^4$ | 0.46 | 10.76 | 2.41 | 0.46 |
| 4 | 0.85 | $4.85\cdot10^4$ | 2.54 | 10.46 | 1.83 | 3.47 |
| 5 | 0.57 | $9.74\cdot10^5$ | 0.87 | 19.05 | 6.1 | 0.81 |
| 6 | 1.06 | $8.48\cdot10^4$ | 2.24 | 12.84 | 2.55 | 2.21 |
| 7 | 3.5 | $8.12\cdot10^3$ | 2.68 | 9.02 | 0.57 | 1.88 |
| 8 | 1.19 | $2.37\cdot10^4$ | 1.81 | 10.34 | 1.69 | 1.36 |
| 9 | 3.1 | $8.37\cdot10^3$ | 0.57 | $7.43\cdot10^3$ | $2.69\cdot10^3$ | 0.57 |
| 10 | 3.3 | $1.08\cdot10^4$ | 1.44 | $9.39\cdot10^3$ | $3.04\cdot10^3$ | 1.53 |
| 11 | 3.09 | $1.2\cdot10^4$ | 1.52 | $1.01\cdot10^4$ | $3.31\cdot10^3$ | 1.49 |
| 12 | 2.8 | $1.38\cdot10^4$ | 2.14 | $1.11\cdot10^4$ | $3.82\cdot10^3$ | 2.08 |
| 13 | 4.25 | $1.03\cdot10^4$ | 1.66 | $1.04\cdot10^4$ | $3.49\cdot10^3$ | 1.60 |
| 14 | 2.19 | $1.87\cdot10^4$ | 6.18 | 10.12 | 1.01 | 6.33 |
| 15 | 0.77 | $1.09\cdot10^5$ | 1.55 | 11.43 | 2.14 | 1.60 |
| 16 | 2.44 | $1.53\cdot10^4$ | 1.15 | $1.06\cdot10^3$ | $3.48\cdot10^3$ | 1.12 |
| 17 | 1.75 | $3.21\cdot10^4$ | 2.09 | $1.44\cdot10^4$ | $5.24\cdot10^3$ | 1.71 |
| 18 | 1.12 | $5.58\cdot10^4$ | 3.73 | 11.3 | 1.86 | 3.76 |
| 19 | 2.69 | $1.52\cdot10^4$ | 1.74 | $1.22\cdot10^4$ | $4.39\cdot10^3$ | 1.66 |
| 20 | 4.16 | $8.64\cdot10^3$ | 0.97 | $7.8\cdot10^3$ | $2.15\cdot10^3$ | 0.96 |
| 21 | 1.58 | $2.12\cdot10^4$ | 1.60 | $9.63\cdot10^3$ | $3.60\cdot10^3$ | 1.34 |
| 22 | 2.62 | $1.48\cdot10^4$ | 1.57 | $1.15\cdot10^4$ | $4.01\cdot10^3$ | 1.46 |
| 23 | 0.37 | $8.78\cdot10^6$ | 2.00 | 17.85 | 6.07 | 1.95 |
| 24 | 0.58 | $8\cdot10^4$ | 0.66 | 16.22 | 4.66 | 0.64 |
| 25 | 0.63 | $7.49\cdot10^4$ | 0.89 | 16.12 | 4.31 | 0.85 |
| 26 | 1.49 | $3.59\cdot10^4$ | 3.60 | $1.5\cdot10^4$ | $5.89\cdot10^3$ | 3.19 |
| 27 | 0.89 | $1.87\cdot10^4$ | 2.54 | 13.26 | 2.69 | 1.37 |
| 28 | 0.41 | $4.46\cdot10^7$ | 2.16 | 21.49 | 6.55 | 2.12 |
| 29 | 0.8 | $3.82\cdot10^4$ | 5.88 | 14.46 | 3.2 | 5.73 |
| 30 | 1.86 | $2.14\cdot10^4$ | 3.39 | $9.0\cdot10^3$ | $2.99\cdot10^3$ | 3.28 |
| 31 | 0.57 | $7.49\cdot10^6$ | 3.05 | 19.47 | 5.02 | 2.95 |
| 32 | 0.87 | $5.16\cdot10^4$ | 4.60 | 15.1 | 3.14 | 4.91 |
| 33 | 2.57 | $1.16\cdot10^4$ | 0.71 | 9.3 | 0.63 | 0.71 |
| 34 | 4.02 | $1.14\cdot10^4$ | 3.35 | $1.01\cdot10^4$ | $2.81\cdot10^3$ | 3.0 |

Since the normal or lognormal distribution does not vary in shape, estimates made assuming a normal or lognormal distribution may be closer to the true values (Figures 10, 11, 14, 15). It is also observed that the selection of the failure – time distributions not only depends on the performance of failure, but it also depends on the sample sizes. If the sample size is large, the Weibull distribution provides a more accurate estimate (Figures 3, 4, 7, 8). The infant mortality is known to follow a Weibull distribution (Figure 3). In certain cases the Weibull distribution is very similar to the normal or lognormal distribution. For a single sample, several distributions will be hypothesized and tested. As a result, it is often the case that more than one distribution will have an acceptable least-squares fit or will pass a goodness-of-fit test. The engineer must select die best distribution from among the acceptable distributions.

The parameters of such distribution function can be properly determined based on the given data and the availability of a failure – time distribution. The method of maximum likelihood provides an efficient and unbiased estimator of the distribution parameters. We compare the theoretical and observed results that the theoretical and observed results match well.

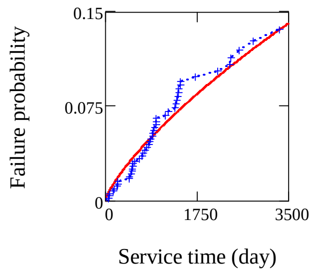


**Figure 1:** Calculated and measured failure probabilities for the $N_2$ leakage in the pressure control switch with Weibull distribution

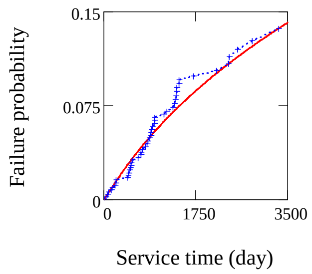


**Figure 2:** Calculated and measured failure probabilities for the $N_2$ leakage in the pressure control switch with Lognormal distribution

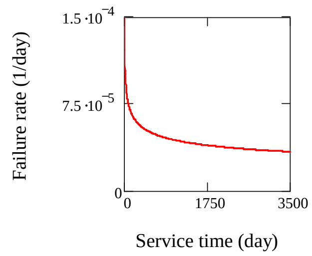


**Figure 3:** Calculated failure rate for the $N_2$ leakage in the pressure control switch with Weibull distribution

All these distributions have been used effectively to analyze lifetime data in the reliability analysis. The results of the analysis are used to estimate the failure rate, the remaining lifetime and the number of failures in coming years. Base on the analysis it can be concluded from Figures 7, 11 and 15 that in the all population of circuit breakers, 65% of the failures are a result of aging in the steep slope of wear-out region of the bathtub curve. The other failure rates of circuit breakers have the decreasing trends and the longer remaining lifetime whereas 70% of decreasing failure rates are involved in control- and protect system (Figure 3). For the mechanical defect of high pressure accumulator, pneumatophore in oil circuit and oil leakage of parallel capacitor it can be seen that the failure rate is relatively stable in coming years. The hydraulic valve and hydraulic circuit, air compressor (Figures 5 to 8) as well as count switcher are the components with the highest contribution of failures and failures can be expected in coming years. The $SF_6$ leakage of circuit breakers (Figures 9 and 12) and the overheat of primary terminal (Figures 13 and 16) shall also be paid attention due to their high failure rates. They show the most rapid change of the remaining lifetime at the same service time. It is recognized from Table I that the leakage is the primary reason to reduce the lifetime of the circuit breakers which lifetime is not longer than 30 years. Statistical methods, also the simple ones, can be applied to support important maintenance decisions, using the data obtained from the failure survey.

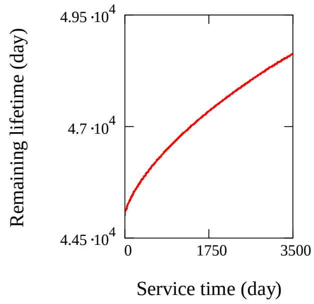


**Figure 4:** Calculated remaining lifetime for the $N_2$ leakage in the pressure control switch with Weibull distribution

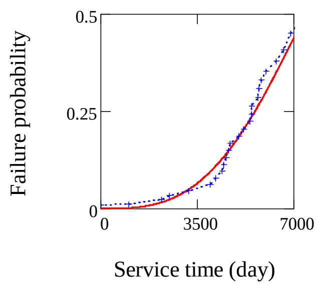


**Figure 5:** Calculated and measured failure probabilities for the fault of pump in the air compressor with Weibull distribution

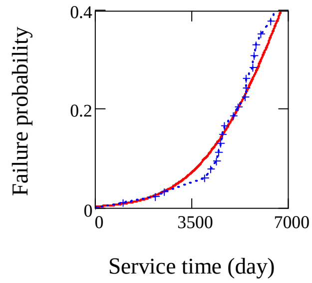


**Figure 6:** Calculated and measured failure probabilities for the fault of pump in the air compressor with Normal distribution

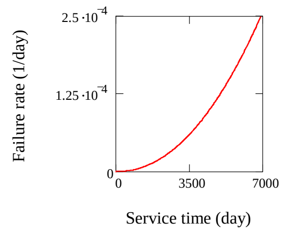


**Figure 7:** Calculated failure rate for the fault of pump in the air compressor with Weibull distribution

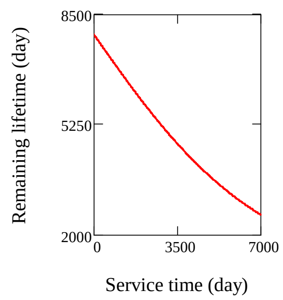


**Figure 8:** Calculated remaining lifetime for the fault of pump in the air compressor with Weibull distribution

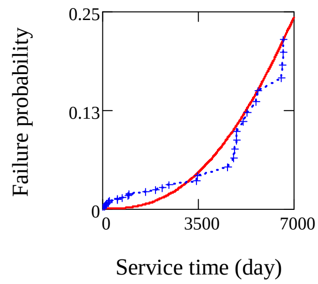


**Figure 9:** Calculated and measured failure probabilities for the $SF_6$ leakage in the circuit breaker with Weibull distribution

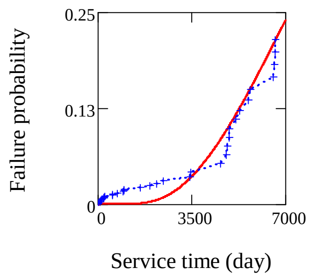


**Figure 10:** Calculated and measured failure probabilities for the $SF_6$ leakage in the circuit breaker with Lognormal distribution

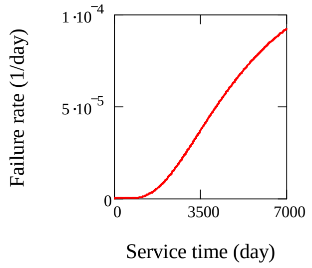


**Figure 11:** Calculated failure rate for the $SF_6$ leakage in the circuit breaker with Lognormal distribution

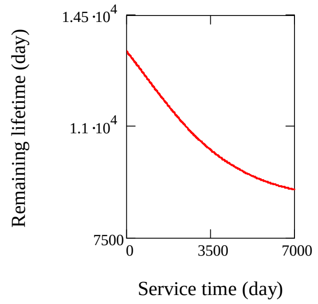


**Figure 12:** Calculated remaining lifetime for the $SF_6$ leakage in the circuit breaker with Lognormal distribution

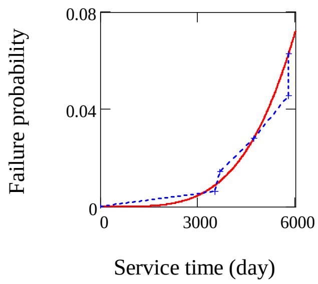


**Figure 13:** Calculated and measured failure probabilities for the overheat of primary terminal in the circuit breaker with Weibull distribution

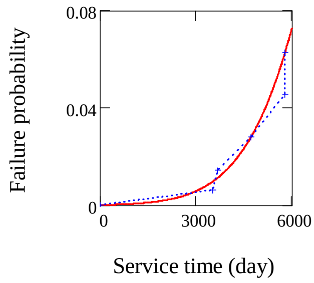


**Figure 14:** Calculated and measured failure probabilities for the overheat of primary terminal in the circuit breaker with Normal distribution

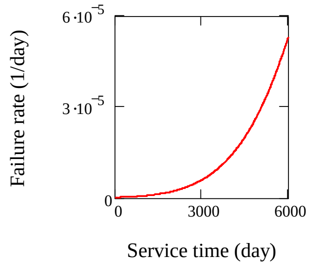


**Figure 15:** Calculated failure rate for the overheat of primary terminal in the circuit breaker with Normal distribution

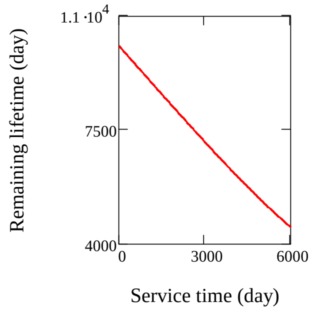


**Figure 16:** Calculated remaining lifetime for the overheat of primary terminal in the circuit breaker with Normal distribution

Infant mortality does not mean “components that fail within 90 days” or any other defined time period. Infant mortality is the time over which the failure rate of a component is decreasing, and may last for years. Electronic components in the protection- and control system as well as pressure control switch, unlike mechanical assemblies, rarely have wear-out mechanisms. Failures during infant mortality are highly undesirable and are always caused by (1) component doesn’t meet requirement; (2) poor design; (3) lack of quality in manufacturing; (4) component installed incorrectly; (5) component constantly stopping and starting; (6) power surges; (7) operator not starting up component according to standard operating procedure. As infant mortality begins to take effect, the prediction of lifetime exists a large derivation as shown in Table II and the reliability of circuit breaker is becoming less controllable. To truly reduce the likelihood of infant mortality and improve reliability, those issues must be addressed and prioritized by risk.

Conversely, wear-out will not always happen long after the expected component life. It is a period when the failure rate is increasing, and has been observed in components after just a few months of use. This, of course, is a disaster from a warranty standpoint! For many mechanical- and thermal defects such as the fault of pump, the $SF_6$ leakage and the overheating of primary terminals, the wear-out time will be shorter than the desired operational life of the whole circuit breaker and replacement of failed assemblies can be used to extend the operational life of the circuit breaker. With some items, wear-out is expected and replacement is a normal routine. In designing a circuit breaker, the engineer must assure that the shortest-lived component lasts long enough to provide a useful service life. If the component is easily replaced, such as relays, replacement may be expected and will not degrade the perception of the circuit breaker's reliability. If the component is not easily replaced and not expected to fail, failure will cause customer dissatisfaction.

## 4 CONCLUSION

In this paper, a statistical approach to investigate the reliability of high voltage $SF_6$ circuit breakers was developed by use of the multiple censoring data. With reference to the equipment’s age, the models can describe the intrinsic ageing of the circuit breakers by means of the maintenance-free data. Furthermore, the goodness-of-fit test and the maximum likelihood method were given to select the probability distributions and to estimate the model parameter subjected to the observation.

## 5 REFERENCES

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[2] X. Zhang, E. Gockenbach, “Life Management of the large power Transformers and the high Voltage $SF_6$ Circuit Breakers”, ISEI 2010, San Diego, CA, USA, pp. 1-5, 2010