MODELLING OF POSITIVE CORONA INCEPTION IN THE COAXIAL CYLINDRICAL ELECTRODE ARRANGEMENT UNDER VARIABLE AIR DENSITY

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Abstract: Corona discharge has attracted much interest among researchers as it has a lot of applications in industry and a number of effects on power systems. Corona inception in the coaxial cylindrical electrode arrangement, which finds many practical applications, depends on the electric field strength around the inner conductor and on atmospheric conditions. In the present study a new model for positive corona inception in a steady or slowly-varying electric field is presented. The model, implementing streamer criterion, assumes that an equivalent electron avalanche develops towards the anode by virtue of ionization by collision and photoionization in an electric field distorted by the avalanche space charge. Based on a great amount of literature experimental data, a new empirical expression for the estimation of positive corona inception field strength has been derived. The formulation of a photoionization coefficient as a function of inner conductor radius and relative air density enabled the investigation of the effects of the latter parameters on the basic characteristics of the avalanche at corona inception, namely critical avalanche length, radius, number and electron density.

1 INTRODUCTION

Corona discharge is of great importance in electrical engineering because of its abundance of industrial applications and effects in power systems. Extensive experimental and theoretical research has focused on corona discharges in the coaxial cylindrical electrode arrangement, which, having the advantage of a known geometric field distribution, finds many practical applications. Therefore, a model for the estimation of corona properties at inception as influenced by inner conductor radius and/or relative air density is useful from, besides fundamental, an engineering point of view.

In a steady or slowly-varying electric field, corona inception occurs when the electric field strength at the inner conductor surface attains a critical value, called corona inception field strength. The latter depends on inner conductor radius and relative air density and is commonly estimated with the aid of empirical expressions, such as the well-known Peek's formula, or through a theoretical approach. In a previous work [1], the computed positive corona inception field strength was found to be in very good agreement with literature experimental data for an avalanche number of about 10^4 when considering solely ionization by collision during avalanche growth.

Although the computational method presented in [1] is a useful tool for engineering applications, it is well established that avalanche growth at positive corona inception is assisted by photoionization in air. In the present study a new model for positive corona inception in the coaxial cylindrical electrode arrangement in air is presented. The model, implementing streamer criterion, assumes that an equivalent electron avalanche develops towards the anode by virtue of ionization by collision and photoionization in an electric field distorted by the avalanche space charge. A new empirical expression for the estimation of positive corona inception field strength has been derived. Also, a photoionization coefficient has been formulated as a function of inner conductor radius and relative air density. Thus, the effects of the latter parameters on the basic characteristics of the critical avalanche at corona inception were investigated.

2 CORONA INCEPTION FIELD STRENGTH

A great amount of experimental results on positive corona inception voltage in the coaxial cylindrical electrode arrangement has been reported in literature [2-21]. It is well established that inner conductor radius and relative air density affect the corona inception field strength; the latter is usually estimated through Peek's empirical formula [2], which can be written as:

$$E_i / \delta = A \cdot \left[1 + \frac{B}{(\delta r_0)^C} \right] \tag{1}$$

where r_0 is the radius of the inner conductor, A, B and C are factors, varying among researchers [1], and δ is the relative air density, given as [22]:

$$\delta = \frac{P}{760} \cdot \frac{293}{273 + T} \tag{2}$$

where P (Torrs) and T (^{o}C) are the pressure and temperature, respectively.

A quadratic expression for the estimation of the positive corona inception field strength in the same electrode arrangement is given in [21]:

$$\left(\frac{E_c}{\delta}\right)^2 - 2\left(\frac{E_c}{\delta}\right)\mathrm{E}_0 \ln\left[\frac{1}{E_0}\left(\frac{E_c}{\delta}\right)\right] - E_0^2 = \frac{K/C}{(\delta r_0)} \quad (3)$$

where K/C = 42 kV^2/cm and E_0 = 24.36 kV/cm.

Expression (1), with factors A, B and C according to Peek [2] as adapted in [21], yields results in good agreement with literature experimental data over the range of 0.0004 < δr_0 < 2 cm; the same is true for (3), however for δr_0 > 0.002 cm (Figure 1). It is noteworthy that for δr_0 smaller than ~0.0004 cm both (1) and (3) largely underestimate the positive corona inception field strength (Figure 1).

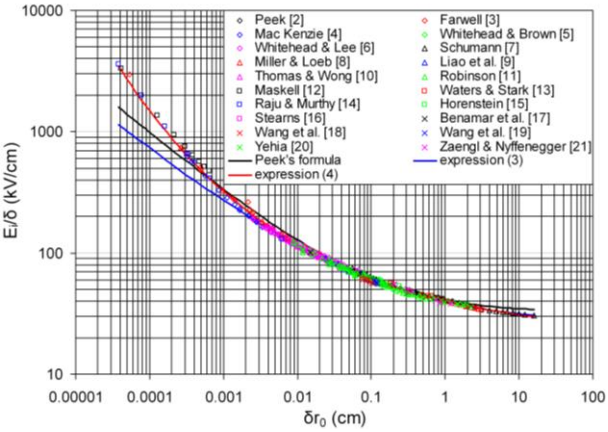

Figure 1: Variation of positive corona inception field strength, E_i/δ, with δr_0, in the coaxial cylindrical electrode arrangement in air.

Based on the literature experimental data shown in Figure 1, a new empirical formula for the estimation of positive corona inception field strength in the coaxial cylindrical electrode arrangement in air has been derived:

$$E_i/\delta = 6.4 + 12.4 \cdot \exp\left(\delta r_0\right)^{-0.17}. \quad (4)$$

The results of (4) are consistent with experimental data for 0.00004 < δr_0 < 16 cm, with an average error over 464 measured values of 2.87%.

3 STREAMER CORONA INCEPTION MODEL

The spatial distribution of the geometric electric field strength in the coaxial cylindrical electrode arrangement is given as:

$$E(r) = \frac{V}{r \cdot \ln\left(R/r_0\right)} \quad \text{for } r_0 \le r \le R \quad (5)$$

where V is the applied voltage, r is the distance from the centre of the inner conductor and r_0 and R is the radius of the inner and outer conductor, respectively (Figure 2).

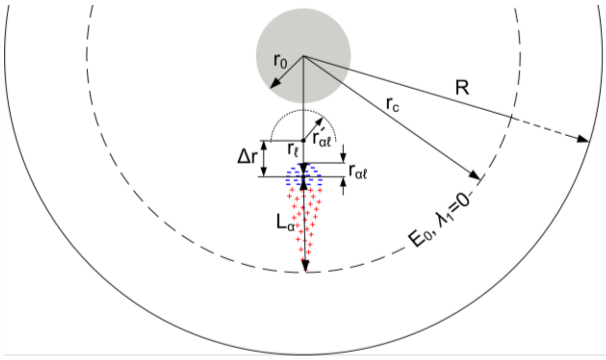

Figure 2: Avalanche formation in the positively stressed coaxial cylindrical electrode arrangement.

If the electric field strength in the gap exceeds a critical value, E_0, sufficient to sustain ionization ("critical field strength"), an electron avalanche may initiate developing towards the inner conductor from a point at a distance r_c from the centre of the inner conductor, where the electric field strength reduces to E_0 or, equivalently, the effective ionization coefficient, λ_1, becomes zero (Figure 2).

According to Hartmann [23], the field dependent effective ionization coefficient, λ_1, ($\lambda_1 = \alpha - \eta + \xi$ with α, η and ξ the first ionization, attachment and detachment coefficients, respectively), is given as:

$$\frac{\lambda_1}{P_0} = M \cdot \left\{ A \cdot \left[1 + \frac{C}{N \cdot \left(E/P_0\right)^3} \right] \cdot e^{\left(\frac{-B \cdot P_0}{E}\right)} - O \cdot \Psi \right\} \quad (6)$$

where $A = 1.75 \cdot 10^3$, $B = 4 \cdot 10^4$, $C = 1.15 \cdot 10^{12}$,

$$M = 1 + 10^{-2} H, \quad N = 1 + 3.2 \cdot 10^{-2} H,$$

$$O = 1 + 1.15 \cdot 10^{-1} H^{0.1}, \quad \Psi = \frac{0.9}{1.49 + \exp\left(-P_0/587\right)}.$$

E is the electric field strength in V/m, P_0 is the pressure in Torrs at 0 ^{o}C and H is the absolute humidity in g/m^3. At standard atmospheric conditions (P= 760Torrs, T= 20^{o}C and H= 11g/m^3) the critical field strength E_0 (λ_1 = 0) is 25.9 kV/cm.

The space charge of the avalanche affects its own growth by distorting the geometric field; the electric field is enhanced in front of the avalanche. Assuming that the space charge at the head of the avalanche is concentrated within a spherical volume, the space charge field associated with an avalanche progressed up to the distance r_ℓ from the centre of the inner conductor is given as:

$$E_s(r) = \frac{Q_{r_\ell} \cdot e}{4\pi\varepsilon_0 \cdot \left(r_\ell - r\right)^2} \quad (7)$$

where e is the electron charge ($1.602\text{x}10^{-19}$ C) and ε_0 is the permittivity of free space ($8.854\text{x}10^{-12}$ F/m). Q_{r_ℓ} is the avalanche number, that is, the number of electrons in the head of the avalanche progressed up to the distance r_ℓ, given as:

$$Q_{r_\ell} = \exp\left(\int_{r_\ell}^{r_c} \lambda_1(r)dr\right) \tag{8}$$

when taking into account solely ionization by collision during avalanche growth. As detailed in [1], under this assumption positive corona inception occurs for an avalanche number of about 10^4 for a wide range of inner conductor radius and relative air density.

However, for avalanche numbers of about 10^4 the associated space charge field is far too low for the fulfilment of streamer criterion. According to the latter, at threshold corona inception conditions an electron avalanche develops into streamer, when the space charge field becomes of the same order of the geometric field [24, 25]. It is well established that positive streamer corona inception occurs rather through a multiavalanche process [26-29], assisted by subsidiary avalanches produced mainly by photoionization in air [16, 26-35]. Hence, to account for the assisting effect of photoionization on positive streamer corona inception, a photoionization coefficient, k_{ph}, is introduced in (8), which then becomes:

$$Q_{r_\ell} = \exp\left(\int_{r_\ell}^{r_c} \left(k_{ph}+1\right)\cdot\lambda_1(r)dr\right). \tag{9}$$

By assuming that the avalanche develops towards the anode in steps of Δr (Figure 2) under the influence of the total electric field, that is, the sum of the geometric and space charge field, given by (5) and (7) respectively, the avalanche number at the distance $r_\ell - \Delta r$ from the centre of the inner conductor is given as:

$$Q_{r_\ell-\Delta r} = Q_{r_\ell}\exp\left(\int_{r_\ell-\Delta r-r'_{a_\ell}}^{r_\ell-r_{a\ell}} \left(k_{ph}+1\right)\cdot\lambda_1(r)dr\right) \tag{10}$$

where Q_{r_ℓ} is given by (9), r_{α_ℓ} and r_{α_ℓ}' are the radii of the head of the avalanche progressed up to the distance r_ℓ and $r_\ell - \Delta r$, respectively. The avalanche radius may be given as [36]:

$$r_{a_\ell} = \sqrt{6\int_{r_\ell}^{r_c}\frac{D}{v_e}dr} \tag{11}$$

where D and v_{e} are the electron transverse diffusion coefficient and electron drift velocity, respectively; the ratio D/v_{e} can be described, as adapted in [23] from [37], as:

$$\frac{D}{v_e} = \frac{3.33\cdot10^{-2}\left(E/P_0\right)^{0.515}}{E}. \tag{12}$$

By using (4) and adopting streamer criterion, the photoionization coefficient k_{ph} in (10) has been estimated through an iterative process employing approximately 10000 steps of Δr (Figure 2). k_{ph} decreases with increasing inner conductor radius, r_0, and especially with relative air density, δ. These effects may well be described by the following empirical expression:

$$k_{ph} = 0.6\cdot r_0^{-0.07}\cdot\delta^{-\left(0.17\cdot r_0^{-0.05}\right)}. \tag{13}$$

The photoionization coefficient according to (13) may be considered as an average value during avalanche growth. Its decrease with relative air density may be associated with the decrease of the mean free path of the emitted photons [38-40].

4 RESULTS AND DISCUSSION

The basic characteristics of the critical avalanche at streamer corona inception, namely length, radius, avalanche number and electron density, were obtained for a wide range of inner conductor radius under variable relative air density and for an absolute humidity of 11 g/m^3.

Figure 3 shows the critical avalanche length, L_{cr}, and radius, r_{cr}, as a function of relative air density, δ, with inner conductor radius, r_0, as parameter. A smaller avalanche, both in length and radius, is required to meet streamer criterion, thus for corona inception, with decreasing r_0 but increasing δ. The shorter length of the critical avalanche can be attributed to the higher geometric field values in the vicinity of the inner conductor, resulting from either a smaller inner conductor radius or a higher

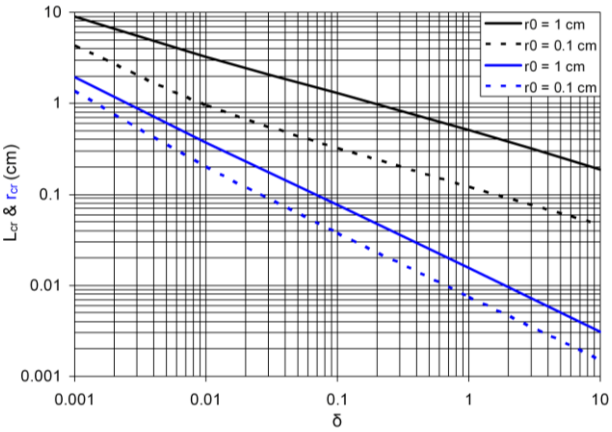

Figure 3: Critical avalanche length, L_{cr}, and radius, r_{cr}, as a function of relative air density δ, with inner conductor radius, r_0, as parameter.

applied field; the latter is required for corona inception due to the increase of the critical field strength with relative air density. The shorter critical avalanche length together with the reduction of the ratio D/v_e with increasing applied field or relative air density may explain, as can be deduced from (11) and (12), the decreasing critical avalanche radius with decreasing inner conductor radius but increasing relative air density. A similar effect of relative air density on critical avalanche length and radius has also been reported in [41].

These results can also be deduced from Figure 4, where the computed critical avalanche length and radius, corresponding to the measured positive corona inception fields shown in Figure 1, are plotted as a function of the product δr_0. Obviously, there is a very good agreement with computed [13, 23, 32, 42] or experimentally observed [43] avalanche length results referring to $\delta = 1$.

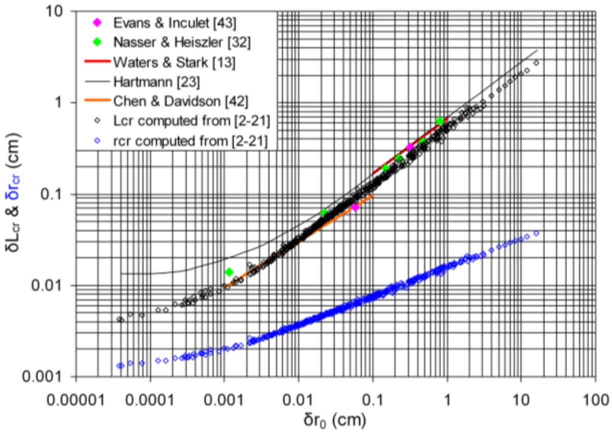

Figure 4: Variation of δL_{cr} and δr_{cr} with δr_0; positive corona inception in the coaxial cylindrical electrode arrangement.

Figure 5 shows the critical avalanche number, Q_{cr}, as a function of relative air density, with inner conductor radius as parameter. A smaller avalanche number is required for corona inception with increasing relative air density, whereas the effect of inner conductor radius depends on relative air density. According to (7), the avalanche number is proportional to the square of avalanche radius; thus, with increasing relative air density, although for the fulfilment of streamer criterion a higher applied field is required therefore also a higher space charge field, the reduction of critical avalanche radius (Figure 3) results in smaller critical avalanche number. A similar effect of relative air density on critical avalanche number has also been reported in [41].

The computed critical avalanche number, corresponding to the measured corona inception fields shown in Figure 1, is plotted against δr_0 in Figure 6. The product δQ_{cr} takes values between 7.5×10^6 and 3×10^8 in a wide range of inner conductor radius and relative air density. The critical avalanche number at standard atmospheric

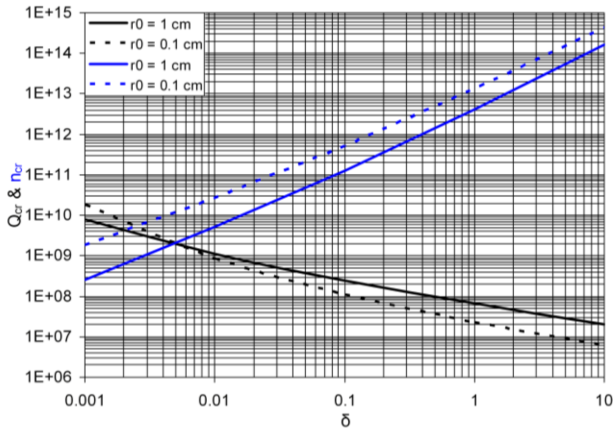

Figure 5: Critical avalanche number, Q_{cr}, and electron density, n_{cr}, as a function of relative air density, δ, with inner conductor radius, r_0, as parameter.

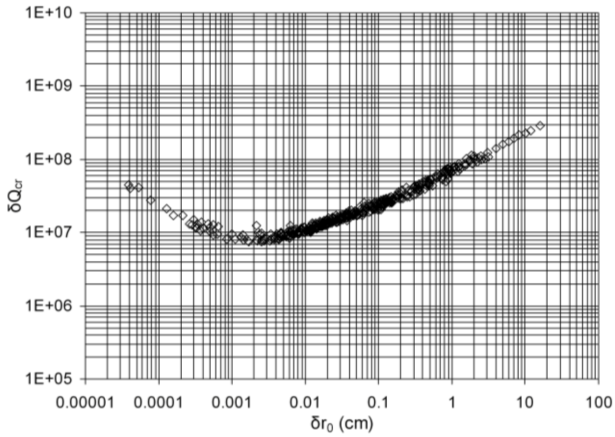

Figure 6: Variation of δQ_{cr} with δr_0; positive corona inception in the coaxial cylindrical electrode arrangement.

conditions is consistent with the generally accepted value of ~10^8 for streamer formation in air [24, 25].

Figure 5 also shows the variation of critical electron density, n_{cr}, calculated with the aid of (14), as a function of relative air density, with inner conductor radius as parameter.

$$n_{cr} = \frac{Q_{cr}}{4/3\,\pi r_{cr}^{3}} \tag{14}$$

Obviously, n_{cr} augments as the inner conductor radius decreases and also with increasing relative air density; this is reasonably attributed to the variation of the critical avalanche radius (Figures 3 and 4). These results can also be deduced from Figure 7, where n_{cr}/δ^2 is plotted as a function of δr_0. Corona inception at standard atmospheric conditions occurs with a critical electron density between 1.35×10^{12} and 4.50×10^{15}; these values are consistent with previously reported values for streamer propagation in air [34, 44].

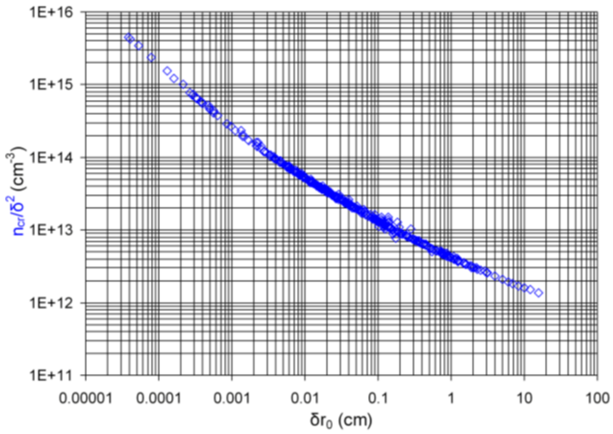

Figure 7: Variation of n_{cr}/δ^2 with δr_0; positive corona inception in the coaxial cylindrical electrode arrangement.

5 CONCLUSIONS

A new model for positive corona inception in the coaxial cylindrical electrode arrangement in air has been presented. The model, implementing streamer criterion, assumes that an equivalent electron avalanche develops towards the anode by virtue of ionization by collision and photoionization in an electric field distorted by the avalanche space charge.

Based on a great amount of literature experimental data, a new empirical expression for the positive corona inception field strength in the coaxial cylindrical electrode arrangement in air has been derived. It is valid (average error < 3%) for values of the product of relative air density and inner conductor radius from 0.00004 cm up to 16 cm. A photoionization coefficient has been formulated as a function of inner conductor radius and relative air density. Thus, the effects of the latter parameters on the basic characteristics of the critical avalanche at positive corona inception have been investigated.

A smaller avalanche, both in length and radius, is required for positive corona inception with decreasing inner conductor radius but increasing relative air density. The critical avalanche number, decreasing with relative air density, is within 7.5×10^{6} and 3×10^{8} at standard atmospheric conditions for a wide range of inner conductor radius. The critical electron density, increasing with relative air density, decreases with inner conductor radius, taking values within 1.35×10^{12} and 4.50×10^{15} at standard atmospheric conditions.

6 ACKNOWLEDGMENT

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7 REFERENCES

[1] P. N. Mikropoulos, V. N. Zagkanas, "A computational method for positive corona inception in the coaxial cylindrical electrode arrangement in air under variable atmospheric conditions", in Proc. 16th Int. Symp. High Voltage Eng., Cape Town, South Africa, paper No. B-10, p. 75, 2009

[2] F. W. Peek: "Dielectric phenomena in HV engineering", New York: McGraw-Hill, 1929

[3] S. P. Farwell, "The corona produced by continuous potentials", AIEE Trans., Vol. 33, pp. 1631-1671, 1914

[4] D. MacKenzie, "The corona in air at continuous potentials and pressures lower than atmospheric", Phys. Rev., Vol. 5, pp. 294-310, 1915

[5] J. B. Whitehead, W. S. Brown, "The electric strength of air.-VII", AIEE Trans., Vol. 36, pp.169-205, 1917

[6] J. B. Whitehead, F. W. Lee, "The electric strength of air under continuous potentials and as influenced by temperature", AIEE Trans., Vol. 40, pp. 1201-1308, 1921

[7] W. O. Schumann, "Uber das minimum der Durchbruchfeldstarke bei Kugelelectroden", Archiv fur Elektrotechnik, Vol. 12, No. 6–12, pp. 593–608, 1923

[8] C. G. Miller, L. B. Loeb, "Positive coaxial cylindrical corona discharges in pure N_2, O_2, and mixtures thereof", J. Appl. Phys., Vol. 22, No. 4, pp. 1226-1230, 1951

[9] T. W. Liao, W. A. Keen, D. R. Powell, "Relationship between corona and radio influence on transmission lines, laboratory studies. I–Point and conductor corona", AIEE Trans. Power App. Syst., Part III, 1957

[10] J. B. Thomas, E. Wong, "Experimental study of dc corona at high temperatures and pressures", J. Appl. Phys., Vol. 29, No. 8, pp. 1226-1230, 1958

[11] M. Robinson, "The corona threshold for coaxial cylinders in air at high pressures", IEEE Trans. Power App. Syst., Vol. PAS-86, No. 2, pp. 185-189, 1967

[12] B. R. Maskell, "The effect of humidity on a corona discharge in air", Royal Aircraft Establishment, Farnborough (UK), Technical Report 70106, 1970

[13] R. T. Waters, W. B. Stark, "Characteristics of the stabilized glow discharge in air", J. Phys. D: Appl. Phys., Vol. 8, pp. 416-426, 1975

[14]G. R. G. Raju, G. R. G. Murthy, “Wire-cylinder corona discharge with the wire at positive potential”, Int. J. Electron., Vol. 46, No. 5, pp. 497-506, 1979

[15]M. N. Horenstein, “Computation of corona space charge, electric field, and V-I characteristic using equipotential charge shells”, IEEE Trans. Indust. Appl., Vol. IA-20, No. 6, 1984

[16]R. G. Stearns, “The positive corona in air: A simplified analytic approach”, J. Appl. Phys., Vol. 66, No. 7, pp. 2899-2913, 1989

[17]B. Benamar, E. Favre, A. Donnot, M. O. Rigo, “Finite element solution for ionized fields in DC electrostatic precipitator”, in Proc. COMSOL Users Conf., Grenoble, France, 2007

[18]W. Wang, C. Li, J. Fan, C. Gu, Y. Jiang, G. Cui, “Study of UHV DC corona performance in a mini corona cage”, in Proc. Electr. Insul. Conf., pp. 459-462, 2007

[19]W. Wang, C. Li, J. Fan, C. Gu, J. Zhou, Y. Jiang, “The effect of temperature and humidity on corona performance of UHV DC transmission line”, IEEE Int. Symp. on Electr. Insul., Vancouver, BC, pp. 66-68, 2008

[20]A. Yehia, “Operating regimes of corona and silent discharges in coaxial cylindrical reactors”, J. Appl. Phys., Vol. 103, No. 7, p. 073301, 2008

[21]E. Kuffel, W. S. Zaengl, J. Kuffel, “High Voltage Engineering: Fundamentals”, Oxford: Newnes, p. 344, 2000

[22]IEC 60060-1, “High-voltage test techniques - Part 1: General definitions and requirements”, 2010

[23]G. Hartmann, “Theoretical evaluation of Peek’s Law”, IEEE Trans. Indust. Appl., Vol. 20, No. 6, pp. 1647-1651, 1984

[24]J. M. Meek, “A theory of spark discharge”, Physics Rev., Vol. 57, pp. 722-728, 1940

[25]H. Raether, Z. Phys. 117, p. 375, 1941

[26]L. B. Loeb, J. M. Meek, “The mechanism of spark discharge in air at atmospheric pressure. I”, J. Appl. Phys., Vol.11, No. 6, pp. 438-447, 1940

[27]L. B. Loeb, R. A. Wijsman, “The theoretical criterion for streamer advance in an electrical field”, J. Appl. Phys., Vol.19, No. 8, pp. 797-799, 1948

[28]I. W. McAllister, G. C. Crichton, E. Bregnsbo, “Experimental study on the onset of positive corona in atmospheric air”, J. Appl. Phys., Vol.50, No. 11, pp. 6797-6805, 1979

[29]M. Laan, P. Paris, “The multi-avalanche nature of streamer formation in inhomogeneous fields”, J. Phys. D: Appl. Phys., Vol. 27, pp. 970-978, 1994

[30]G. A. Dawson, W. P. Winn, “A model for streamer propagation”, Z. Phys., Vol. 183, No. 2, pp. 159-171, 1965

[31]L. E. Kline, J. G. Siambis, “Computer simulation of electrical breakdown in gases; avalanche and streamer formation”, Phys. Rev. A, Vol. 5, No. 2, pp. 794-805, 1972

[32]E. Nasser and M. Heiszler, “Mathematical-physical model of the streamer in nonuniform fields”, J. Appl. Phys., Vol. 45, No. 8, 1974

[33]G. R. G. Raju, J. Liu, “Simulation of electrical discharges in gases. Nonuniform electric fields”, IEEE Trans. Dielectr. Electr. Insul., Vol. 2, No. 5, pp. 1016-1041, 1995

[34]R. Morrow, J. J. Lowke, “Streamer propagation in air”, J. Phys. D: Appl. Phys., Vol. 30, No. 4, pp. 614-627, 1997

[35]L. Arevalo, M. Beccera, F. Roman, “Discharge geometry behavior in a positive corona simulation in a non-uniform coaxial arrangement”, in Proc. 5th Int. Conf. on Power Systems and Electromagnetic Compatibility, Corfu, Greece, pp. 106-110, 2005

[36]R. C. Fletcher, “Impulse breakdown in the 10^{-9}-sec. range of air at atmospheric pressure”, Phys. Rev., Vol. 76, No. 10, pp. 1501-1511, 1949

[37]J. Dutton, “A survey of electron swarm data”, J. Phys. Chem. Ref. Data, Vol. 4, pp. 577-856, 1975

[38]T. H. Teich, “Emission gasionisierender Strahlung aus Elektronenlawinen”, Z. Phys. Vol. 199, No. 4, pp. 378-394, 1967

[39]S. Badaloni, I. Gallimberti, “Basic data of air discharges”, Padua University Report, Upee-72/05, 1972

[40]G. W. Penney, G. T. Hummert, “Photoionization Measurements in Air, Oxygen, and Nitrogen”, J. Appl. Phys., Vol. 41, No. 2, pp. 572-577, 1970

[41]N. L. Allen, J. C. P. Kong, “Positive corona inception in air at elevated temperatures”, IEE IEE Proc., Sci. Meas. Technol., Vol. 153, No. 1, pp. 31-38, 2006

[42]J. Chen, J. H. Davidson, “Electron density and energy distributions in the positive DC corona: Interpretation for corona-enhanced chemical reactions”, Plasma Chem. Plasma Process., Vol. 22, No. 2, 2002

[43]R. W. Evans, I. I. Inculet, “The radius of the visible ionization layer for positive and negative coronas”, 1978 IEEE Trans. Indust. Appl., Vol. IA-14, No. 6, 1978

[44]A. A. Kulikovsky, “The role of photoionization in positive streamer dynamics”, J. Phys. D: Appl. Phys., Vol. 33, No. 12, pp. 1514-1524, 2000