MODELLING OF POSITIVE CORONA INCEPTION IN THE COAXIAL CYLINDRICAL ELECTRODE ARRANGEMENT UNDER VARIABLE AIR DENSITY

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Abstract: Corona discharge has attracted much interest among researchers as it has a lot of applications in industry and a number of effects on power systems. Corona inception in the coaxial cylindrical electrode arrangement, which finds many practical applications, depends on the electric field strength around the inner conductor and on atmospheric conditions. In the present study a new model for positive corona inception in a steady or slowly-varying electric field is presented. The model, implementing streamer criterion, assumes that an equivalent electron avalanche develops towards the anode by virtue of ionization by collision and photoionization in an electric field distorted by the avalanche space charge. Based on a great amount of literature experimental data, a new empirical expression for the estimation of positive corona inception field strength has been derived. The formulation of a photoionization coefficient as a function of inner conductor radius and relative air density enabled the investigation of the effects of the latter parameters on the basic characteristics of the avalanche at corona inception, namely critical avalanche length, radius, number and electron density.

1 INTRODUCTION

Corona discharge is of great importance in electrical engineering because of its abundance of industrial applications and effects in power systems. Extensive experimental and theoretical research has focused on corona discharges in the coaxial cylindrical electrode arrangement, which, having the advantage of a known geometric field distribution, finds many practical applications. Therefore, a model for the estimation of corona properties at inception as influenced by inner conductor radius and/or relative air density is useful from, besides fundamental, an engineering point of view.

In a steady or slowly-varying electric field, corona inception occurs when the electric field strength at the inner conductor surface attains a critical value, called corona inception field strength. The latter depends on inner conductor radius and relative air density and is commonly estimated with the aid of empirical expressions, such as the well-known Peek’s formula, or through a theoretical approach. In a previous work [1], the computed positive corona inception field strength was found to be in very good agreement with literature experimental data for an avalanche number of about $10^4$ when considering solely ionization by collision during avalanche growth.

Although the computational method presented in [1] is a useful tool for engineering applications, it is well established that avalanche growth at positive corona inception is assisted by photoionization in air. In the present study a new model for positive corona inception in the coaxial cylindrical electrode arrangement in air is presented. The model, implementing streamer criterion, assumes that an equivalent electron avalanche develops towards the anode by virtue of ionization by collision and photoionization in an electric field distorted by the avalanche space charge. A new empirical expression for the estimation of positive corona inception field strength has been derived. Also, a photoionization coefficient has been formulated as a function of inner conductor radius and relative air density. Thus, the effects of the latter parameters on the basic characteristics of the critical avalanche at corona inception were investigated.

2 CORONA INCEPTION FIELD STRENGTH

A great amount of experimental results on positive corona inception voltage in the coaxial cylindrical electrode arrangement has been reported in literature [2-21]. It is well established that inner conductor radius and relative air density affect the corona inception field strength; the latter is usually estimated through Peek’s empirical formula [2], which can be written as:

$$E_i \delta = A \left[ 1 + \frac{B}{(\delta r_0)} \right]^{C}$$  

(1)

where $r_0$ is the radius of the inner conductor, $A$, $B$ and $C$ are factors, varying among researchers [1], and $\delta$ is the relative air density, given as [22]:

$$\delta = \frac{P}{760} \frac{293}{273 + T}$$  

(2)
where \( P \) (Torr) and \( T \) (°C) are the pressure and temperature, respectively.

A quadratic expression for the estimation of the positive corona inception field strength in the same electrode arrangement is given in [21]:

\[
\left(\frac{E_c}{\delta}\right)^2 - 2\left(\frac{E_c}{\delta}\right)E_0 \ln \left[ \frac{1}{E_0} \left(\frac{E_c}{\delta}\right) \right] - E_0 = \frac{K/C}{(\delta_0)}
\] (3)

where \( K/C = 42 \text{kV}^2/\text{cm} \) and \( E_0 = 24.36 \text{kV/cm} \).

Expression (1), with factors \( A, B \) and \( C \) according to Peek [2] as adapted in [21], yields results in good agreement with literature experimental data over the range of \( 0.0004 < \delta_0 < 2 \text{ cm} \); the same is true for (3), however for \( \delta_0 > 0.002 \text{ cm} \) (Figure 1). It is noteworthy that for \( \delta_0 \) smaller than \( \sim 0.0004 \text{ cm} \) both (1) and (3) largely underestimate the positive corona inception field strength (Figure 1).

**Figure 1:** Variation of positive corona inception field strength, \( E/\delta \), with \( \delta_0 \), in the coaxial cylindrical electrode arrangement in air.

Based on the literature experimental data shown in Figure 1, a new empirical formula for the estimation of positive corona inception field strength in the coaxial cylindrical electrode arrangement in air has been derived:

\[
\frac{E}{\delta} = 6.4 + 12.4 \cdot \exp(\delta_0^{-0.17}).
\] (4)

The results of (4) are consistent with experimental data for \( 0.00004 < \delta_0 < 16 \text{ cm} \), with an average error over 464 measured values of 2.87%.

### 3 STREAMER CORONA INCEPTION MODEL

The spatial distribution of the geometric electric field strength in the coaxial cylindrical electrode arrangement is given as:

\[
E(r) = \frac{V}{r \cdot \ln(R/r_0)} \text{ for } r_0 \leq r \leq R
\] (5)

where \( V \) is the applied voltage, \( r \) is the distance from the centre of the inner conductor and \( r_0 \) and \( R \) is the radius of the inner and outer conductor, respectively (Figure 2).

**Figure 2:** Avalanche formation in the positively stressed coaxial cylindrical electrode arrangement.

If the electric field strength in the gap exceeds a critical value, \( E_0 \), sufficient to sustain ionization ("critical field strength"), an electron avalanche may initiate developing towards the inner conductor from a point at a distance \( r_i \) from the centre of the inner conductor, where the electric field strength reduces to \( E_0 \) or, equivalently, the effective ionization coefficient, \( \lambda_1 \), becomes zero (Figure 2).

According to Hartmann [23], the field dependent effective ionization coefficient, \( \lambda_1 \), \( \lambda_1 = \alpha - \eta + \xi \) with \( \alpha, \eta \) and \( \xi \) the first ionization, attachment and detachment coefficients, respectively, is given as:

\[
\frac{\lambda_1}{P_0} = M \cdot \left[ 1 + \frac{C}{N \left( \frac{E}{P_0} \right)^{1/2}} \right] e^{-B/P_0^{1/2}} - O \cdot \Psi
\] (6)

where \( A = 1.75 \cdot 10^7, B = 4 \cdot 10^4, C = 1.15 \cdot 10^{12}, M = 1 + 10^{-3} H, N = 1 + 3.2 \cdot 10^{-2} H, O = 1 + 1.15 \cdot 10^{-9} H^{0.1}, \Psi = \frac{0.9}{1.49 + \exp(-P_0/587)} \).

\( E \) is the electric field strength in \( \text{V/m} \), \( P_0 \) is the pressure in Torr at \( 0 \text{ °C} \) and \( H \) is the absolute humidity in \( \text{g/m}^3 \). At standard atmospheric conditions (\( P_0 = 760 \text{ Torr}, T = 20 \text{ °C} \) and \( H = 11g/m^3 \)) the critical field strength \( E_0(\lambda_1 = 0) \) is 25.9 \text{kV/cm}.

The space charge of the avalanche affects its own growth by distorting the geometric field; the electric field is enhanced in front of the avalanche. Assuming that the space charge at the head of the avalanche is concentrated within a spherical volume, the space charge field associated with an avalanche progressed up to the distance \( r_i \) from the centre of the inner conductor is given as:

\[
E_i(r) = \frac{Q_i \cdot e}{4 \pi \varepsilon_0 \cdot ((r_i - r)^2)}
\] (7)
where $e$ is the electron charge ($1.602 \times 10^{-19}$ C) and $\varepsilon_0$ is the permittivity of free space ($8.854 \times 10^{-12}$ F/m). $Q_\ell$ is the avalanche number, that is, the number of electrons in the head of the avalanche progressed up to the distance $r_\ell$ given as:

$$Q_\ell = \exp \left( \int_{r_\ell} r \lambda_e (r) dr \right)$$

(8)

when taking into account solely ionization by collision during avalanche growth. As detailed in [1], under this assumption positive corona inception occurs for an avalanche number of about $10^4$ for a wide range of inner conductor radius and relative air density.

However, for avalanche numbers of about $10^4$ the associated space charge field is far too low for the fulfillment of streamer criterion. According to the latter, at threshold corona inception conditions an electron avalanche develops into streamer, when the space charge field becomes of the same order of the geometric field [24, 25]. It is well established that positive streamer corona inception occurs rather through a multiavalanche process [26-29], assisted by subsidiary avalanches produced mainly by photoionization in air [16, 26-35]. Hence, to account for the assisting effect of photoionization on positive streamer corona inception, a photoionization coefficient, $k_{ph}$, is introduced in (8), which then becomes:

$$Q_\ell = \exp \left( \int_{r_\ell} \left( k_{ph} + 1 \right) \lambda_e (r) dr \right)$$

(9)

By assuming that the avalanche develops towards the anode in steps of $\Delta r$ (Figure 2) under the influence of the total electric field, that is, the sum of the geometric and space charge field, given by (5) and (7) respectively, the avalanche number at the distance $r_\ell - \Delta r$ from the centre of the inner conductor is given as:

$$Q_{r_\ell - \Delta r} = Q_\ell \exp \left( \int_{r_\ell - \Delta r}^{r_\ell} \left( k_{ph} + 1 \right) \lambda_e (r) dr \right)$$

(10)

where $Q_\ell$ is given by (9), $r_{a\ell}$ and $r_{\ell - \Delta r}$ are the radii of the head of the avalanche progressed up to the distance $r_\ell$ and $r_\ell - \Delta r$, respectively. The avalanche radius may be given as [36]:

$$r_{a\ell} = \sqrt{\frac{\int_{r_\ell} r D v_e dr}{v_e}}$$

(11)

diffusion coefficient and electron drift velocity, respectively; the ratio $D/v_e$ can be described, as adapted in [23] from [37], as:

$$\frac{D}{v_e} = 3.33 \times 10^{-2} \left( \frac{E}{P_0} \right)^{0.515}$$

(12)

By using (4) and adopting streamer criterion, the photoionization coefficient $k_{ph}$ in (10) has been estimated through an iterative process employing approximately 10000 steps of $\Delta r$ (Figure 2). $k_{ph}$ decreases with increasing inner conductor radius, $r_0$, and especially with relative air density, $\delta$. These effects may well be described by the following empirical expression:

$$k_{ph} = 0.6 \cdot n_0^{-0.07} \cdot \delta^{-0.17 \cdot n_0^{-0.05}}$$

(13)

The photoionization coefficient according to (13) may be considered as an average value during avalanche growth. Its decrease with relative air density may be associated with the decrease of the mean free path of the emitted photons [38-40].

4 RESULTS AND DISCUSSION

The basic characteristics of the critical avalanche at streamer corona inception, namely length, radius, avalanche number and electron density, were obtained for a wide range of inner conductor radius under variable relative air density and for an absolute humidity of $11 \text{ g/m}^3$.

Figure 3 shows the critical avalanche length, $L_{cr}$, and radius, $r_{cr}$, as a function of relative air density, $\delta$, with inner conductor radius, $r_0$, as parameter. A smaller avalanche, both in length and radius, is required to meet streamer criterion, thus for corona inception, with decreasing $r_0$ but increasing $\delta$. The shorter length of the critical avalanche can be attributed to the higher geometric field values in the vicinity of the inner conductor, resulting from either a smaller inner conductor radius or a higher

![Graph](image-url)
applied field; the latter is required for corona inception due to the increase of the critical field strength with relative air density. The shorter critical avalanche length together with the reduction of the ratio $D/\nu_e$ with increasing applied field or relative air density may explain, as can be deduced from (11) and (12), the decreasing critical avalanche radius with decreasing inner conductor radius but increasing relative air density. A similar effect of relative air density on critical avalanche length and radius has also been reported in [41].

These results can also be deduced from Figure 4, where the computed critical avalanche length and radius, corresponding to the measured positive corona inception fields shown in Figure 1, are plotted as a function of the product $\delta r_0$. Obviously, there is a very good agreement with computed [13, 23, 32, 42] or experimentally observed [43] avalanche length results referring to $\delta = 1$.

Figure 5 shows the critical avalanche number, $Q_{cr}$, as a function of relative air density, with inner conductor radius as parameter. A smaller avalanche number is required for corona inception with increasing relative air density, whereas the effect of inner conductor radius depends on relative air density. According to (7), the avalanche number is proportional to the square of avalanche radius; thus, with increasing relative air density, although for the fulfilment of streamer criterion a higher applied field is required therefore also a higher space charge field, the reduction of critical avalanche radius (Figure 3) results in smaller critical avalanche number. A similar effect of relative air density on critical avalanche number has also been reported in [41].

The computed critical avalanche number, corresponding to the measured corona inception fields shown in Figure 1, is plotted against $\delta r_0$ in Figure 6. The product $\delta Q_{cr}$ takes values between $7.5 \times 10^6$ and $3 \times 10^8$ in a wide range of inner conductor radius and relative air density. The critical avalanche number at standard atmospheric conditions is consistent with the generally accepted value of $\sim 10^8$ for streamer formation in air [24, 25].

Figure 5 also shows the variation of critical electron density, $n_{cr}$, calculated with the aid of (14), as a function of relative air density, with inner conductor radius as parameter.

$$n_{cr} = \frac{Q_{cr}}{4/3 \pi r_{cr}^3}$$

Obviously, $n_{cr}$ augments as the inner conductor radius decreases and also with increasing relative air density; this is reasonably attributed to the variation of the critical avalanche radius (Figures 3 and 4). These results can also be deduced from Figure 7, where $n_{cr}/\delta^3$ is plotted as a function of $\delta r_0$. Corona inception at standard atmospheric conditions occurs with a critical electron density between $1.35 \times 10^{12}$ and $4.50 \times 10^{15}$; these values are consistent with previously reported values for streamer propagation in air [34, 44].
5 CONCLUSIONS

A new model for positive corona inception in the coaxial cylindrical electrode arrangement in air has been presented. The model, implementing streamer criterion, assumes that an equivalent electron avalanche develops towards the anode by virtue of ionization by collision and photoionization in an electric field distorted by the avalanche space charge.

Based on a great amount of literature experimental data, a new empirical expression for the positive corona inception field strength in the coaxial cylindrical electrode arrangement in air has been derived. It is valid (average error < 3%) for values of the product of relative air density and inner conductor radius from 0.00004 cm up to 16 cm. A photoionization coefficient has been formulated as a function of inner conductor radius and relative air density. Thus, the effects of the latter parameters on the basic characteristics of the critical avalanche at positive corona inception have been investigated.

A smaller avalanche, both in length and radius, is required for positive corona inception with decreasing inner conductor radius but increasing relative air density. The critical avalanche number, decreasing with relative air density, is within 7.5x10^6 and 3x10^8 at standard atmospheric conditions for a wide range of inner conductor radius. The critical electron density, increasing with relative air density, decreases with inner conductor radius, taking values within 1.35x10^{12} and 4.50x10^{15} at standard atmospheric conditions.

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