MEASURING AND MODELING OF BALANCED AND UNBALANCED SINGLE-CIRCUIT LINES

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Abstract: Accurate parameters of a line are very important for protection settings (distance protection relays and fault locators) and power system modelling. Power system calculations and protection settings are often based on balanced line models. However, most operational overhead transmission lines or HV cables are unbalanced. Different models for unbalanced and balanced lines and the associated symbols are presented since confusion of parameters has to be avoided. The conversions between the models are stated. The paper proposes a practical method to determine the symmetrical components of an unbalanced single-circuit by means of performing measurements. The measurement procedure, equipment used and associated calculations to compute symmetrical components are described. Different methods to calculate the line impedances from the measurements are in detail discussed and analyzed. A case study is presented and the results are assessed.

1 INTRODUCTION

In order to accurately parameterize the individual distance protection equipment, particularly with regard to single-phase faults, the exact line impedance values for both the positive sequence system and the zero sequence system are required [1]. The k-factor (determined by the positive sequence impedance and the ground impedance matching factor) resulting from these values is essential for accurate settings.

In the past, zero sequence impedances were calculated based on geometrical assumptions. This method never proved capable of providing settings which allowed the protection to reliably detect single-phase faults, resulting in an unacceptable amount of incorrect tripping [2]. Moreover, the inaccurately displayed fault locations resulting from the over-reach or under-reach of the protection also made it difficult for network control personnel to direct repair teams to the exact location where the fault occurred.

One important setting of a distance protection relay is the k-factor. The impedance of a phase-to-ground fault is different from a phase-to-phase fault. Because the impedance of the ground path, or to be more precise, of the line-to-ground loop, is different, a factor within the relay gives the relation between the line-to-line and the line-to-ground impedances. This factor has many names [3], it is called ground impedance matching factor, residual compensation factor, earthing factor or simply k-factor, as it is often referred to.

Accurately measured zero sequence impedances allow reliable and correct operation of distance protection and fault locators. Both different line models with different parameters and different protection equipment with different settings have to be considered. Accurately measured impedance values are achieved by avoiding parameter confusion and by correctly evaluating the measurement results.

2 MODELS OF BALANCED AND UNBALANCED LINES

There are different models for unbalanced and balanced lines. It is necessary to introduce the used models and the used symbols. The models are presented in this section. Shunt admittances are neglected as it is normally done for short circuit calculation and distance protection.

2.1 Physical Model of an Unbalanced Line

Figure 1 shows the equivalent circuit diagram of an unbalanced line. The model in Figure 1 is described by its equation

$$\Delta\underline{U}_L = \underline{Z}_L \underline{I}_L \quad \text{respectively}$$

$$\begin{pmatrix} \Delta\underline{U}_{L1} \\ \Delta\underline{U}_{L2} \\ \Delta\underline{U}_{L3} \end{pmatrix} = \begin{pmatrix} \underline{Z}_{L11} & \underline{Z}_{L12} & \underline{Z}_{L13} \\ \underline{Z}_{L21} & \underline{Z}_{L22} & \underline{Z}_{L23} \\ \underline{Z}_{L31} & \underline{Z}_{L32} & \underline{Z}_{L33} \end{pmatrix} \begin{pmatrix} \underline{I}_{L1} \\ \underline{I}_{L2} \\ \underline{I}_{L3} \end{pmatrix} \tag{1}$$

where $\underline{I}_L$ is the vector of currents, $\Delta\underline{U}_L$ is the vector of longitudinal voltages and $\underline{Z}_L$ is the impedance matrix of the single-circuit line.

The impedance matrix $\underline{Z}_L$ is symmetric

$$\underline{Z}_{Lik} = \underline{Z}_{Lki} \tag{2}$$

The model of an unbalanced line is determined by six parameters

- three diagonal elements $\underline{Z}_{L11}$, $\underline{Z}_{L22}$ and $\underline{Z}_{L33}$ and

- three non-diagonal elements $\underline{Z}_{L12}$, $\underline{Z}_{L13}$ and $\underline{Z}_{L23}$.

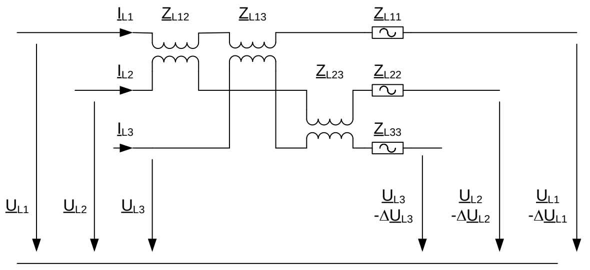


Figure 1: Equivalent circuit diagram of a single-circuit line containing only series impedances

2.2 Symmetrical Components of an Unbalanced Line

The impedance matrix $\underline{Z}_L$ can be transferred in symmetrical components using the transformation matrix $\underline{T}$

$$\underline{T} = \frac{1}{3}\begin{pmatrix} 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & 1 & 1 \end{pmatrix} \tag{3}$$

The physical components are transformed in symmetrical components with

$$\Delta\underline{U}_s := \underline{T}\Delta\underline{U}_L,\ \underline{Z}_s := \underline{T}\underline{Z}_L\underline{T}^{-1} \text{ and } \underline{I}_s := \underline{T}\underline{I}_L \tag{4}$$

The model can also be described by its equation

$$\Delta\underline{U}_s = \underline{Z}_s\underline{I}_s$$

$$\begin{pmatrix} \Delta\underline{U}_{s1} \\ \Delta\underline{U}_{s2} \\ \Delta\underline{U}_{s0} \end{pmatrix} = \begin{pmatrix} \underline{Z}_{s11} & \underline{Z}_{s12} & \underline{Z}_{s10} \\ \underline{Z}_{s21} & \underline{Z}_{s22} & \underline{Z}_{s20} \\ \underline{Z}_{s01} & \underline{Z}_{s02} & \underline{Z}_{s00} \end{pmatrix}\begin{pmatrix} \underline{I}_{s1} \\ \underline{I}_{s2} \\ \underline{I}_{s0} \end{pmatrix} \tag{5}$$

The model of an unbalanced line in symmetrical components is determined by six parameters

- three diagonal elements $\underline{Z}_{s11}$, $\underline{Z}_{s22}$ and $\underline{Z}_{s00}$ and
- three non-diagonal elements $\underline{Z}_{s12}$, $\underline{Z}_{s10}$ and $\underline{Z}_{s20}$.

The diagonal elements $\underline{Z}_{s11}$ and $\underline{Z}_{s22}$ are equal

$$\underline{Z}_{s11} = \underline{Z}_{s22} \tag{6}$$

2.3 Physical Model of a Balanced Line

The entries of the impedance matrix of a balanced line meet the conditions

$$\underline{Z}_{L12} = \underline{Z}_{L13} = \underline{Z}_{L23} \quad \text{and} \quad \underline{Z}_{L11} = \underline{Z}_{L22} = \underline{Z}_{L33} \tag{7}$$

The model of a balanced line is determined by two parameters

- the diagonal element $\underline{Z}_{L11}$ and
- the non-diagonal element $\underline{Z}_{L12}$.

2.4 Symmetrical Components of a Balanced Line

The impedance matrix of the balanced single-circuit line in symmetrical components is uncoupled

$$\underline{Z}_s = \begin{pmatrix} \underline{Z}_{s11} & 0 & 0 \\ 0 & \underline{Z}_{s11} & 0 \\ 0 & 0 & \underline{Z}_{s00} \end{pmatrix} \tag{8}$$

Figure 2 shows the model of a balanced line in symmetrical components.

The model of a balanced line in symmetrical components is determined by two parameters

- the positive sequence impedance $\underline{Z}_{s11}$ (equal to the negative sequence impedance $\underline{Z}_{s22}$) and
- the zero sequence impedance $\underline{Z}_{s00}$.

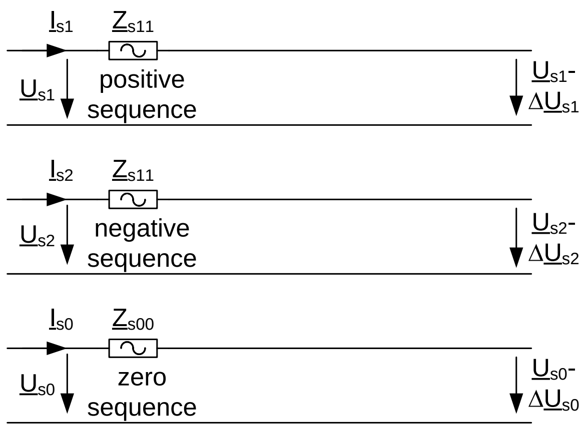


Figure 2: Equivalent circuit diagram of a balanced line in symmetrical components

2.5 Balanced Line Model with Earth Return Impedance

Figure 3 shows the model with earth return impedance of a balanced line. The balanced line model with earth return impedance is determined by two parameters

- the element of phases $\underline{Z}_p$ and
- the element of earth $\underline{Z}_E$.

It should be mentioned that this model should only be applied for balanced line modelling. It is not useful to assign the phase elements different values, like

$$\underline{Z}_{p1} \neq \underline{Z}_{p2} \neq \underline{Z}_{p3} \tag{9}$$

in order to model unbalanced lines. The differences of the phase elements only influence the diagonal elements of the physical model (see equation (1)). Additionally, a model with four parameters is created instead of the models in section 2.1 and 2.2, which both contain six parameters.

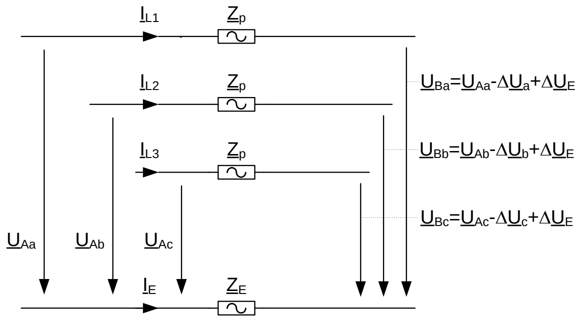


Figure 3: Equivalent circuit diagram of the line model with earth return impedance

3 CONVERSION OF MODEL PARAMETERS

The equations to convert the parameters of the different models are specified in this section.

The equation to transfer the unbalanced models is already specified in equation (4).

3.1 Conversion Between Balanced Line Models

The three balanced models presented in sections 2.3, 2.4 and 2.5 are completely equivalent according to the boundary condition

$$-\underline{I}_E = \underline{I}_{L1} + \underline{I}_{L2} + \underline{I}_{L3} \tag{9}$$

The three models are defined by two parameters. These parameters can be converted with

$$\underline{Z}_{s11} = \underline{Z}_{L11} - \underline{Z}_{L12} = \underline{Z}_p \tag{10}$$

and

$$\underline{Z}_{s00} = \underline{Z}_{L11} + 2\underline{Z}_{L12} = \underline{Z}_p + 3\underline{Z}_E \tag{11}$$

3.2 Conversion from Unbalanced to Balanced Model

Overhead lines and cables are normally unbalanced but short circuit calculation and distance protection are using the balanced models. Therefore, a conversion from unbalanced to balanced models is needed.

Using equation (4) it can be shown that

$$\underline{Z}_{s11} = \underline{Z}_p = \overline{\underline{Z}_{Lii}} - \overline{\underline{Z}_{Lik}} \tag{12}$$

and

$$\underline{Z}_{s00} = \underline{Z}_p + 3\underline{Z}_E = \overline{\underline{Z}_{Lii}} + 2\overline{\underline{Z}_{Lik}} \tag{13}$$

where

- $\overline{\underline{Z}_{Lii}}$ is the arithmetic mean value of the diagonal elements of the unbalanced physical model and
- $\overline{\underline{Z}_{Lik}}$ is the arithmetic mean value of the non-diagonal elements of the unbalanced physical model.

Equation (12) and (13) are important on a measurement point of view. They point out that the parameters of a balanced line model can be determined by calculation of the arithmetic mean value using the measurement values of an unbalanced line.

4 MEASUREMENT OF LINE IMPEDANCES

It is necessary to measure the line parameters because parameter calculations normally are not reliable as already pointed out.

4.1 Measurement of Line Parameters

With a single-phase source seven measurements can be performed (see section 5.3). Every unbalanced model is described by six parameters; every balanced model is described by two parameters. This means there are measurements left for a plausibility check.

Table 1 shows the measured loops, the measuring results and the determined parameters for an unbalanced line. Equation (12) and (13) are used to determine the parameters of a balanced line model.

Table 1: Measured loops, measuring results and determined parameter

measured loop	measuring result	determined parameter
L1-E	$\frac{\underline{U}_m}{\underline{I}_m} = \frac{\underline{U}_{L1}}{\underline{I}_{L1}} = \underline{Z}_{L11}$	$\underline{Z}_{L11}$
L2-E	$\frac{\underline{U}_m}{\underline{I}_m} = \frac{\underline{U}_{L2}}{\underline{I}_{L2}} = \underline{Z}_{L22}$	$\underline{Z}_{L22}$
L3-E	$\frac{\underline{U}_m}{\underline{I}_m} = \frac{\underline{U}_{L3}}{\underline{I}_{L3}} = \underline{Z}_{L33}$	$\underline{Z}_{L33}$
L1-L2	$\frac{\underline{U}_m}{\underline{I}_m} = \frac{\Delta\underline{U}_{L1} - \Delta\underline{U}_{L2}}{\underline{I}_{L1}}$ $= \underline{Z}_{L11} + \underline{Z}_{L22} - 2\underline{Z}_{L12}$	$\underline{Z}_{L12}$
L1-L3	$\frac{\underline{U}_m}{\underline{I}_m} = \frac{\Delta\underline{U}_{L1} - \Delta\underline{U}_{L3}}{\underline{I}_{L1}}$ $= \underline{Z}_{L11} + \underline{Z}_{L33} - 2\underline{Z}_{L13}$	$\underline{Z}_{L13}$

L2-L3	$\frac{\underline{U}_{m}}{\underline{I}_{m}} = \frac{\Delta\underline{U}_{L2} - \Delta\underline{U}_{L3}}{\underline{I}_{L2}} = \underline{Z}_{L22} + \underline{Z}_{L33} - 2\underline{Z}_{L23}$	$\underline{Z}_{L23}$
"U_0-$3I_0$"	$\frac{\underline{U}_{m}}{\underline{I}_{m}} = \frac{\Delta\underline{U}_{s0}}{3\underline{I}_{s0}} = \frac{1}{3}\underline{Z}_{s00}$	$\underline{Z}_{s00}$

4.2 Calculations and Modern Measurement Equipment

Compared to calculations, measurements of line parameters including the k-factors are relatively straightforward, when using modern measurement equipment.

Historically, the problem when measuring line parameters was to overcome disturbances and interferences from other live systems. Therefore, either currents close to or above the nominal current had to be used. Alternatively, big diesel generators were needed to allow the beat method to be applied.

Today, electronic generators allow the use of signals with frequencies shifted away from the mains. By applying a frequency-selective measurement that measures only the part of the incoming signal that matches the generated frequency accurate results are delivered - even in an environment that experiences major disturbances. With this method it is possible to work with currents that are a fraction of the nominal line current and consequently the weight of the equipment becomes comparable with components weighing less than 30 kg (Figure 4).

Surge arrestors that are part of the measurement system can safely discharge currents of up to 30 kA to ground. These provide optimum safety during testing in case of an unexpected event on the cable, such as a fault on an adjacent system.

Figure 4 Equipment for line impedance measurement

5 CASE STUDY

5.1 Malfunction of Distance Protection Relay.

A line fault occurred in close vicinity to Substation B on the transmission line between Substation A and B (Figure 5). Both relays at Substation A and Substation C interpreted the fault as ZONE 1 and consequently both relays tripped. The relay at station C consequently overreached and incorrectly tripped the line between Substation B and Substation C.

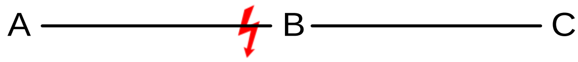


Figure 5 Line diagram indicating the position of the fault.

The affected utility experiences an unnecessary false trip and decided to measure the sequence impedance to check the relay settings.

5.2 Line Data

The 115 kV overhead transmission line, between substation B and C, was 9.5 km long. The line is not transposed and consists of a mixture of vertical and delta tower types, as depicted in (Figure 6). The phase allocations are also indicated in Figure 6.

The k-factor setting of the relay was based on the following simulated sequence impedance data:

$$\underline{k}_{o\_SIM} = \frac{\underline{Z}_{s00\_SIM}}{\underline{Z}_{L\_SIM}}$$
$$= 3.643 \angle -5.23°$$
$$where: \tag{14}$$
$$\underline{Z}_{s11\_SIM} = 0.6213 + 3.9675i$$
$$\underline{Z}_{s00\_SIM} = 3.5713 + 14.1882i$$

5.3 Practical Measurements

In total, seven measurements per system were performed, three for each combination of phase-to-phase loops (Figure 7), three for each phase-to-ground (Figure 8) and one for all three phases-to-ground (Figure 9).

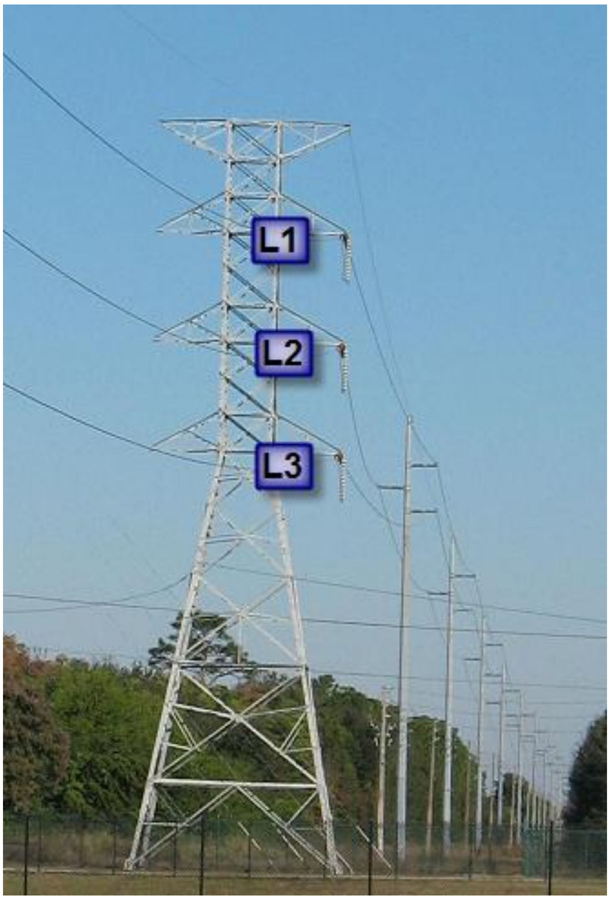
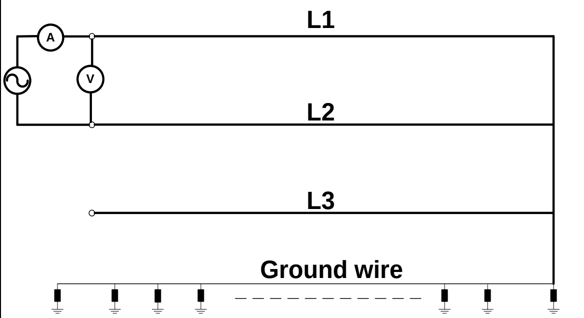
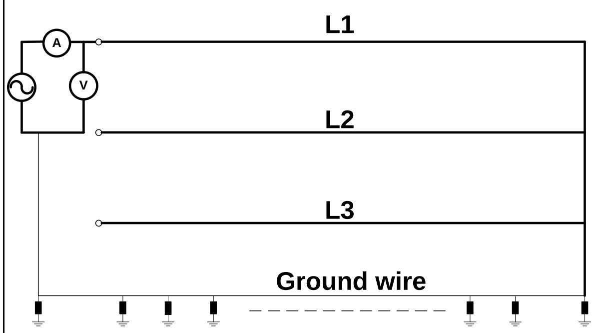


Figure 6 Overhead transmission in case study



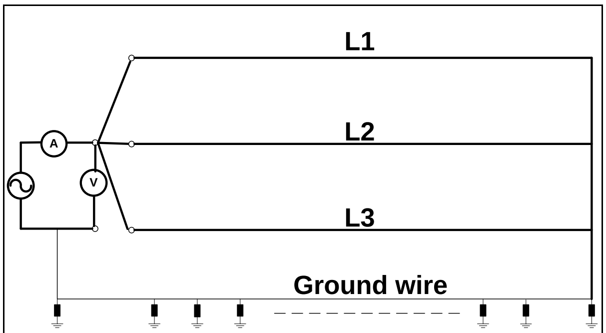
	Measurement results:				
Name	Note:	R [Ω]	X [Ω]	Z [Ω]	Phi (°)
Z_{M1}	Line 1-2:	1.256	7.754	7.855	80.802
Z_{M2}	Line 2-3:	1.324	7.731	7.843	80.282
Z_{M3}	Line 3-1:	1.323	8.350	8.454	80.997

Figure 7 Measurement diagram for the phase-to-phase loop impedance (measurement loop between line 1 and 2) together with all the phase-to-phase measurement results.



	Measurement results:				
Name	Note:	R [Ω]	X [Ω]	Z [Ω]	Phi (°)
Z_{M4}	Line1-E	1.228	5.744	5.873	77.936
Z_{M5}	Line2-E	1.211	5.877	6.001	78.358
Z_{M6}	Line3-E	1.276	5.900	6.037	77.794

Figure 8 Measurement diagram of the phase-to-ground impedance loop (line 1) together with the other phase-to-ground measurement results.



	Measurement results:				
Name	Note:	R [Ω]	X [Ω]	Z [Ω]	Phi (°)
Z_{M7}	Line123-E	0.821	3.190	3.294	75.571

Figure 9 Measurement diagram of line impedance between all three phases in parallel to ground loop, together with the measurement result.

6 PROCESSING THE MEASUREMENT RESULTS

The first three measurements represent the phase-to-phase loop impedances (Figure 7) and from these measurements the phase impedance can be calculated:

$$\underline{Z}_{L1} = \frac{1}{2}\frac{\underline{U}_{LL}}{\underline{I}_{LL}} = \frac{1}{2} Z_{M1} = 0.627 + 4.186i \qquad (15)$$

The measured phase impedance $\underline{Z}_L$ also equals the positive sequence impedance according to equation 12. A summary of all the positive sequence impedance is given in Table 2.

Table 2 Positive sequence impedance and average calculated from the individual phase-to-phase loops.

	R [Ω]	X [Ω]	Z [Ω]	Phi (°)
$Z_{s11(L1)}$	0.627	4.186	4.233	81.479
$Z_{s11(L2)}$	0.628	3.567	3.622	80.011
$Z_{s11(L3)}$	0.696	4.163	4.221	80.514
Z_{s11} (Average)	0.650	3.972	4.025	80.702

From this result, a close correlation can be noted by comparing this measured result (Table 2) with original calculated line data as given in equation (14).

From the next three phase-to-ground loop measurements (Z_{M4} to Z_{M6}), the individual earth impedances and consequently the average earth impedance can be calculated. For phase 1 the earth impedance can be calculated as follows:

$$\underline{Z}_{E\_FROM\_M4} = \left(\frac{\underline{U}_{LE}}{\underline{I}_{LE}} - \underline{Z}_L \right) = \left(Z_{M4} - \underline{Z}_{L1} \right) = 0.6 + 1.557i \qquad (16)$$

By substituting the measured phase impedance and earth impedance values to equation 13 the zero sequence impedance can be calculated. The results are shown in Table 3.

Table 3 Zero sequence impedance and average calculated from the individual phase to ground loops.

	R [Ω]	X [Ω]	Z [Ω]	Phi (°)
$Z_{S00(L1-E)}$	2.428	8.858	9.185	74.670
$Z_{S00\ (L2-E)}$	2.376	10.497	10.763	77.246
$Z_{S00\ (L3-E)}$	2.438	9.374	9.686	75.422
Z_{S00} Average	2.414	9.577	9.876	75.851

From this result a major discrepancy (45% for the real values and 48.24% for the imaginary values) can be noted between the measured and the calculated results. Also from individual zero sequence impedance, a relatively big variation can be noted between the different phases. This gives a clear indication that this transmission line is unbalanced like most practical lines.

Measurement number 7 (Figure 9) is the most direct and accurate way to measure the zero sequence impedance of a line and can be calculated as follows:

$$Z_{s00(M7)} = 3Z_{M7} = 2.463 + 9.571i \qquad (17)$$

It is remarkable to note that the two calculations, which are based on different measurement loops, deliver nearly identical values for the zero sequence impedance when comparing the result obtained from equation 17 with the average value in Table 3.

Finally, the measured k-factors can be expressed as

$$\underline{k}_o = \frac{\underline{Z}_{s00(M7)}}{\underline{Z}_{s11}} = 2.455 \angle -5.13° \qquad (18)$$

This measured k-factor differs 48.4% (in magnitude) from the original k-factor which the utility obtained by using a simulation program.

It was interesting to note that after some detailed investigations, the utility discovered that the change from steel ground wire to optical ground wire (OPGW) was not incorporated in the simulation program. Due to potential errors that can occur by using simulations and calculations, the utility decided to measure the line impedance of all major lines to ensure correct relay settings and thereby avoiding unnecessary false trips.

7 CONCLUSION

Overhead lines and cables are normally unbalanced but short circuit calculation and distance protection are using the balanced models. Therefore, a conversion from unbalanced to balanced models is needed. The different models and conversion between models are described in this paper. A practical case study highlights the importance to measure the sequence impedance since simulations are prone to errors. The case study also shows how practical measurements of an unbalanced line can be used to get the right parameters for distance protection relay settings. The distance protection relay would have operated correctly (not over-reaching) if the settings were based on the measured sequence impedance data at the time of the fault.

8 REFERENCES

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