ESTIMATION OF TIME PARAMETER UNCERTAINTIES DUE TO VOLTAGE UNCERTAINTIES IN LIGHTNING IMPULSE VOLTAGE MEASUREMENT SYSTEMS

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Abstract: The lightning impulse voltage is widely used in testing high voltage equipment. The shape of the impulse generated by an impulse generator is defined by two time parameters, the front time T_1 and the time to half-value T_2. There has been insufficient work on the estimation of uncertainties of the time parameters due to the uncertainties of the voltage measurement, such as those from the voltage non-linearity of the measurement system and digitiser noise. This is, in part, because the sensitivity of the uncertainties of the time parameters to the uncertainties of voltage values that define the time parameters cannot be determined analytically. This paper describes a numerical procedure to determine these sensitivity coefficients and provides examples of calculating time parameter uncertainties from voltage uncertainties using these sensitivity coefficients. The sensitivity coefficients of T_1 and T_2 uncertainties with respect to the voltage uncertainties are found to be constant within the limits of T_1 and T_2 values specified by the relevant IEC standard.

1 INTRODUCTION

IEC 60060-1 [1] provides a definition of the lightning impulse which is commonly used in high voltage tests. The shape of the impulse is specified by two parameters, the front time, T_1, and the time to half-value, T_2. These time parameters are defined from time instants that correspond to a number of voltage levels relative to the impulse peak voltage.

The measurement uncertainties of these voltage values influence the time parameter uncertainties. It is therefore important that when calculating the time parameter uncertainties, the contributions of the voltage uncertainties are taken into account. To estimate the contribution of the voltage uncertainties, however, requires determination of the sensitivity coefficients [2] of the time parameters with respect to their defining voltages. Usually, sensitivity coefficients can be conveniently determined from the analytical model function [2] that describes the relationship between the measurand and its input quantities. However, this is not case with the relationship between the time parameters of the impulse and the voltages that define them, even if the analytical function of the output of an ideal impulse generator is used to represent the impulse.

As described in [1, 3], the relationship between the voltage of an impulse, $v(t)$, and time, t, can be best approximated by the function that describes the output of an ideal impulse generator:

$$v(t) = A(e^{-t/\tau_1} - e^{-t/\tau_2}) \qquad (1)$$

where τ_1, τ_2 and A are constants.

Although the time parameters T_1 and T_2 can be determined numerically from an impulse that is described by (1), the relationship between either T_1 or T_2 and the constants of (1) is irrational [3]. The lack of an analytical relationship makes it difficult to determine the required time parameter sensitivity coefficients and hence the required uncertainty values. This component of time parameter uncertainty is therefore often ignored in practice, although it can be significant.

The aim of this paper is to provide values of the sensitivity coefficients that can be used for uncertainty calculations in practical impulse tests. Numerical calculation is used to determine the sensitivity coefficients of T_1 and T_2 with respect to voltage uncertainties. The sensitivity coefficients are calculated for two common sources of voltage uncertainties in impulse voltage measurement; first, the voltage non-linearity measured by the voltage non-linearity test [4] and second, the digitising noise. To determine the variation of the sensitivity coefficients with the values of the time parameters, the sensitivity coefficients were calculated using a number of waveforms with time parameters covering the tolerance limits specified in IEC 60060-1 [1].

2 MODEL FUCTIONS OF TIME PARAMETERS

The IEC definitions [1] for the lighting impulse voltage define four time instants, t_0, t_{30}, t_{50} and t_{90}. The time instants t_{30} and t_{90} are the time values corresponding to the voltages on the impulse front that are 30% and 90% respectively of the peak voltage. The time instant t_0 is the time value where the straight line through the two voltage levels corresponding to t_{30} and t_{90} intercepts the zero voltage level. Finally t_{50} is the time corresponding to the voltage on the tail of the impulse that is 50%

of the peak voltage. T_1 is then defined as a function of t_{30} and t_{90}, and T_2 as a function of t_{50} and t_0.

In the analysis henceforth, the voltages corresponding to t_0, t_{30}, t_{90} and t_{90} are denoted as v_0, v_{30}, v_{90} and v_{50}, and the peak voltage as v_{100}. The term *defining instant* is used for t_0, t_{30}, t_{90} and t_{50} and the term *defining voltage* for v_0, v_{30}, v_{90}, v_{50} and v_{100}. Finally, the defining instants and the defining voltages together are termed the *defining values* of the time parameters.

The relationships between T_1 and T_2 and their defining instants can be described by:

$$T_1 = 1.67(t_{90} - t_{30}) \tag{2}$$

$$T_2 = t_{50} - t_0 \tag{3}$$

with each of the defining instants being functions of their defining voltages, i.e., :

$$t_{30} = f(v_{30}, v_{100}) \tag{4}$$

$$t_{90} = g(v_{90}, v_{100}) \tag{5}$$

$$t_{50} = h(v_{50}, v_{100}) \tag{6}$$

$$t_0 = k(v_{30}, v_{90}, v_{100}) \tag{7}$$

It is evident from the above definitions that a given set of T_1 and T_2 values does not uniquely define a practical waveform. However, since most practical impulses with the same T_1 and T_2 values differ only slightly in shape, the sensitivity coefficients for a particular pair of T_1 and T_2 values may be estimated from the corresponding ideal impulse described by (1), to a first order approximation. Therefore, although ideal impulses that follow (1) are used in the calculations in this paper, the results of the calculations remain relevant to practical impulses.

3 SENSITIVITY COEFFICIENTS OF TIME PARAMETERS WITH RESPECT TO DEFINING VOLTAGES

The sensitivity coefficients are determined from the partial derivatives of T_1 and T_2 with respect to their defining voltages, v_{30}, v_{90} and v_{50} by a combination of analytical derivation and numerical calculation. The sensitivity coefficients are first calculated from the non-distorted ideal impulse waveforms, and the results are then related to the sensitivity of T_1 and T_2 to voltage errors due to non-linearity distortion or to uncertainties of the defining voltages.

From (2) to (7), the sensitivity coefficients of T_1 with respect to v_{30} and v_{90} and the sensitivity coefficient of T_2 with respect to v_{50}, can be expressed as:

$$\frac{\partial T_1}{\partial v_{30}} = \frac{\partial T_1}{\partial t_{30}} \cdot \frac{\partial t_{30}}{\partial v_{30}} = -1.67 \cdot \frac{\partial t_{30}}{\partial v_{30}} \tag{8}$$

$$\frac{\partial T_1}{\partial v_{90}} = \frac{\partial T_1}{\partial t_{90}} \cdot \frac{\partial t_{90}}{\partial v_{90}} = +1.67 \cdot \frac{\partial t_{90}}{\partial v_{90}} \tag{9}$$

$$\frac{\partial T_2}{\partial v_{50}} = \frac{\partial T_2}{\partial t_{50}} \cdot \frac{\partial t_{50}}{\partial v_{50}} = \frac{\partial t_{50}}{\partial v_{50}} \tag{10}$$

The partial derivatives $\partial t_{30} / \partial v_{30}$, $\partial t_{90} / \partial v_{90}$ and $\partial t_{50} / \partial v_{50}$ can be determined by numerical calculation from the ideal impulse waveforms digitised from (1) by finding the changes in t_{30}, t_{90} and t_{50} caused by the corresponding small changes in v_{30}, v_{90} and v_{50}. These partial derivatives use the time shifts caused by voltage changes around the defining voltages along the non-distorted ideal waveform. The next step is to relate these partial derivatives to the changes of the defining instants caused by small voltage errors at the defining voltages. The errors in the defining voltages cause the waveform to deviate from the original one. However, it can be shown that when the voltage error is sufficiently small, $\partial t_{30} / \partial v_{30}$, $\partial t_{90} / \partial v_{90}$ and $\partial t_{50} / \partial v_{50}$ (of the ideal waveform) can be used to estimate the sensitivity of the errors of the defining instants to errors of the defining voltages.

Let us first consider the effect of voltage errors caused by voltage non-linearity distortion as an example (Figures 1 and 2). We introduce primed variables to describe the distorted waveform while the variables without primes refer to the original (or ideal) waveform. Hence the derivatives

$$\partial t'_{30}/\partial v'_{30},\ \partial t'_{90}/\partial v'_{90} \text{ and } \partial t'_{50}/\partial v'_{50}$$

are used to represent the sensitivity of errors of defining instants to the errors of defining voltages caused by distortion.

Figures 1 and 2 show that a time shift caused by a change of voltage value due to waveform distortion is opposite in sign to the time shift due to the same voltage change on the original waveform. Therefore, when the distortion is small the following relationships hold:

$$\frac{\partial T_1}{\partial v'_{30}} \approx -\frac{\partial T_1}{\partial v_{30}} \tag{11}$$

$$\frac{\partial T_1}{\partial v'_{90}} \approx -\frac{\partial T_1}{\partial v_{90}} \tag{12}$$

$$\frac{\partial T_2}{\partial v'_{50}} \approx -\frac{\partial T_2}{\partial v_{50}} \tag{13}$$

where $\frac{\partial T_1}{\partial v'_{30}}$, $\frac{\partial T_1}{\partial v'_{90}}$, and $\frac{\partial T_2}{\partial v'_{50}}$ are the sensitivity coefficients of T_1 and T_2 with respect to the voltage non-linearity errors at the defining voltages, v_{30}, v_{90} and v_{50}.

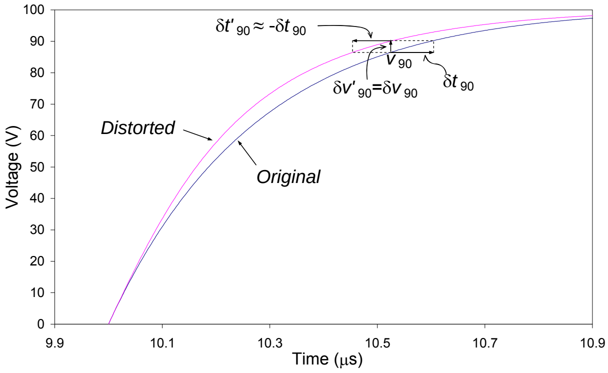

Figure 1 Time shift $\delta t'_{90}$ due to waveform distortion.

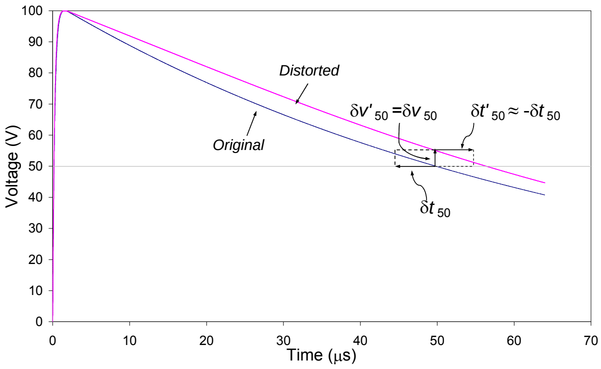

Figure 2 Time shift $\delta t'_{50}$ due to waveform distortion.

By substituting (8), (9) and (10) into (11), (12) and (13) respectively, we have the following relationships:

$$\frac{\partial T_1}{\partial v'_{30}} \approx +1.67 \cdot \left(\frac{\partial t_{30}}{\partial v_{30}}\right) \tag{14}$$

$$\frac{\partial T_1}{\partial v'_{90}} \approx -1.67 \cdot \left(\frac{\partial t_{90}}{\partial v_{90}}\right) \tag{15}$$

$$\frac{\partial T_2}{\partial v'_{50}} \approx -\frac{\partial t_{50}}{\partial v_{50}} \tag{16}$$

i.e., the sensitivity of time parameter errors with respect to voltage distortion errors can be estimated from the partial derivatives of the non-distorted waveforms.

The effect of the error of the defining voltage v_{100} on T_1 and T_2 may be understood by considering (2) to (6). A distortion error in v_{100} will cause errors in v_{30}, v_{90} and v_{50} that are proportional to the error of v_{100}. Changes in these voltage values due to the error of v_{100} will therefore lead to time shifts that are in the *opposite* direction to those of $\partial t'_{30}$, $\partial t'_{90}$ and $\partial t'_{50}$, which are caused by the local distortion errors in v_{30}, v_{90} and v_{50}. Therefore, using (2), (4) and (5), we have:

$$\begin{aligned}
\frac{\partial T_1}{\partial v'_{100}} = \frac{\partial T_1}{\partial v_{100}} &= \frac{\partial T_1}{\partial t_{90}} \cdot \frac{\partial t_{90}}{\partial v_{100}} + \frac{\partial T_1}{\partial t_{30}} \cdot \frac{\partial t_{30}}{\partial v_{100}} \\
&= \frac{\partial T_1}{\partial t_{90}} \cdot \frac{\partial t_{90}}{\partial v_{90}} \cdot \frac{\partial v_{90}}{\partial v_{100}} + \frac{\partial T_1}{\partial t_{30}} \cdot \frac{\partial t_{30}}{\partial v_{30}} \cdot \frac{\partial v_{30}}{\partial v_{100}} \\
&= \frac{\partial T_1}{\partial v_{90}} \cdot \frac{\partial v_{90}}{\partial v_{100}} + \frac{\partial T_1}{\partial v_{30}} \cdot \frac{\partial v_{30}}{\partial v_{100}} \\
&\approx -\frac{\partial T_1}{\partial v'_{90}} \cdot \frac{\partial v_{90}}{\partial v_{100}} - \frac{\partial T_1}{\partial v'_{30}} \cdot \frac{\partial v_{30}}{\partial v_{100}} \\
&\approx -0.9 \cdot \frac{\partial T_1}{\partial v'_{90}} - 0.3 \cdot \frac{\partial T_1}{\partial v'_{30}}
\end{aligned}$$

…(17)

Similarly it can be shown that

$$\frac{\partial T_2}{\partial v'_{100}} \approx -0.5 \cdot \frac{\partial T_2}{\partial v'_{50}} \tag{18}$$

Note that the voltage error is assumed to be small, so that the effect on T_2 of t_0 via changes to v_{30}, v_{90} and v_{100} is negligible, i.e.,

$$\frac{\partial T_2}{\partial v'_{30}} \approx 0, \quad \frac{\partial T_2}{\partial v'_{90}} \approx 0 \text{ and } \frac{\partial T_2}{\partial v'_{100}} \approx 0$$

Table 1 shows the numerical partial derivatives of four non-distorted ideal waveforms covering the tolerance ranges of T_1 and T_2 specified in IEC 60060-1 [1] and the sensitivity coefficients of the time parameter errors with respect to the errors of their defining voltages, calculated using (14) to (18). The calculation was performed using digitised samples of equation (1) with a sample interval of 2 ns, and a peak voltage of 100 V, which was arbitrarily chosen for convenience of analysis.

The values and the signs of the calculated sensitivity coefficients in Table 1 can be understood as follows. Taking the low positive value of $\partial T_1 / \partial v'_{30}$ as an example, a positive error in v_{30} causes t'_{30} to move towards the start of the impulse, and hence increases T_1. This results in a positive error in T_1. On the other hand, the slope near v_{30} is much greater than that near v_{90}, therefore a voltage error at v_{30} causes a much smaller change in T_1 than the same voltage error at v_{90}. The negative value for $\partial T_1 / \partial v'_{90}$ arises because a positive error in v_{90} will also cause t'_{90} to move towards the start of the impulse, which will decrease the T_1 value and hence result in a negative error in T_1.

Table 1 Partial derivatives of ideal waveforms and sensitivity coefficients of T_1 and T_2 errors
(all expressed in μs/V for a peak voltage of 100 V).

Impulse Waveform	0.84/50	1.20/50	1.56/60	1.20/40
$\partial t_{30} / \partial v_{30}$	+0.003 84	+0.005 56	+0.007 23	+0.005 59
$\partial t_{90} / \partial v_{90}$	+0.0242 4	+0.034 1	+0.044 4	+0.033 9
$\partial t_{50} / \partial v_{50}$	−1.400	−1.364	−1.647	−1.077
$\partial T_1 / \partial v'_{30}$	+0.006 40	+0.009 29	+0.012 08	+0.009 33
$\partial T_1 / \partial v'_{90}$	−0.040 5	−0.057 0	−0.074 2	−0.056 6
$\partial T_1 / \partial v'_{100}$	+0.034 5	+0.048 5	+0.062 1	+0.047 3
$\partial T_2 / \partial v'_{50}$	+1.400	+-1.364	+1.647	+1.077
$\partial T_2 / \partial v'_{100}$	−0.700 0	−0.682 0	−0.823 5	−0.538 4

A positive error in v_{100} alone will increase v_{90} and v_{30} and also cause t_{30} and t_{90} to shift away from the start of the impulse. Since the shift in t_{90} is much greater than that of t_{30} for same change in v_{30} and in v_{90}, this will produce a positive T_1 error with its magnitude being equal to the negative of the sum of the T_1 error caused by the v_{90} error independently and the T_1 error caused by the v_{30} error independently.

Similarly, the values of the $\partial T_2 / \partial v'_{50}$ and $\partial T_2 / \partial v'_{100}$ can be explained in terms of the shift of t_{50} caused by errors in v_{50} and v_{100}.

4 CORRELATION OF DEFINING VOLTAGES

The values of v_{30}, v_{90} and v_{50} are correlated to v_{100} because they are defined with reference to it. To calculate the combined standard uncertainty of the time parameters, the correlations between the uncertainties of the correlated defining voltages have to be determined.

Let us first assign the following symbols for the standard absolute uncertainties for the defining voltages:

u_{30} for v_{30}, u_{50} for v_{50}, u_{90} for v_{90} and u_{100} for v_{100}.

The correlation coefficients can be determined using the approximate formula given in C.3.6 NOTE 3 of ISO GUM [2] as:

$$r_{100,30} = \frac{u_{100} \cdot \delta v_{30}}{u_{30} \cdot \delta v_{100}} \tag{19}$$

$$r_{100,90} = \frac{u_{100} \cdot \delta v_{90}}{u_{90} \cdot \delta v_{100}} \tag{20}$$

$$r_{100,50} = \frac{u_{100} \cdot \delta v_{50}}{u_{50} \cdot \delta v_{100}} \tag{21}$$

where δv_{30}, δv_{50} and δv_{90} are small changes in v_{30}, v_{50} and v_{90} caused by a small change in v_{100}, δv_{100}.

We will consider two common practical cases for the values of these correlation coefficients.

Case 1 Voltage non-linearity

Voltage non-linearity measured according to IEC60060-2 [4] usually does not represent the true voltage non-linearity of the measurement system. Rather it is intended to prove that the measured output is linear with the input voltage within the measured non-linearity limit. This is especially true when the non-linearity limit is established by comparing the impulse peak voltage reading and the reading of the generator charging voltage. Furthermore, the measured voltage non-linearity is often a single value expressed as a percentage of the scale factor. For practical impulse tests, it may be converted (by assuming an appropriate statistical distribution) to a single value of relative standard uncertainty [4], u_{rel}, for all voltages, i.e.,

$$u_{rel} = \frac{u_{30}}{v_{30}} = \frac{u_{50}}{v_{50}} = \frac{u_{90}}{v_{50}} = \frac{u_{100}}{v_{100}} \tag{22}$$

In this case, by rearranging (19), (20) and (21), the correlation coefficients can be determined as:

$$r_{100,30} = r_{100,50} = r_{100,90} = 1$$

i.e., when the relative uncertainties of all defining voltages are equal, the correlation coefficients of their absolute uncertainties are all equal to 1.

Case 2 Quantization noise

Quantization noise is usually of a fixed variance at all voltage levels within a given voltage range. The standard uncertainty of the noise voltage is then either expressed in the absolute term, or a percentage of the peak voltage. In this case, since

$$u_{30} = u_{50} = u_{90} = u_{100}$$

from (19) to (21), the values of correlation coefficients are:

$$r_{100,30} = 0.3,\ r_{100,90} = 0.9 \text{ and } r_{100,50} = 0.5$$

5 COMBINED STANDARD UNCERTAINTY OF T_1 AND T_2 DUE TO VOLTAGE UNCERTAINTIES

The partial derivatives and the sensitivity coefficients given in Table 1 can also be applied to random variations although they are derived from errors caused by stable waveform distortion. In other words, they can also be applied to estimation of uncertainties of the time parameters due to uncertainties of the defining voltages.

Uncertainties of all defining voltages will contribute to the uncertainties of the time parameters. In practice, uncertainties in the individual defining voltages are not distinguished from one another. Rather, a single value of voltage uncertainty, either a relative uncertainty, as in Case 1, or an absolute uncertainty, as in Case 2, may be readily estimated and considered sufficient.

Let us use these two common cases again to calculate the combined standard uncertainties of T_1 and T_2. We will continue to use an impulse peak voltage of 100 V, for which we have previously calculated and listed the sensitivity coefficients (refer to Table 1).

For Case 1, we will use an arbitrary value of 0.4% for the relative voltage standard uncertainty, and will use a value of 0.4 V for the absolute voltage standard uncertainty for Case 2.

The combined absolute standard uncertainty of T_1, u_1, is then calculated using the equation for combined standard uncertainty in [2]:

$$u_1 = \sqrt{\left(\frac{\partial T_1}{\partial v'_{30}}\right)^2 u_{30}{}^2 + \left(\frac{\partial T_1}{\partial v'_{90}}\right)^2 u_{90}{}^2 + \left(\frac{\partial T_1}{\partial v'_{100}}\right)^2 u_{100}{}^2 + u_{c1}} \quad \ldots(23)$$

where

$$u_{c1} = 2 \cdot r_{30,100}(\partial T_1/\partial v_{30})(\partial T_1/\partial v_{100})u_{30}u_{100} + 2 \cdot r_{90,100}(\partial T_1/\partial v_{90})(\partial T_1/\partial v_{100})u_{90}u_{100}$$

is the correlation component of u_1.

Similarly, the absolute uncertainty of T_2, u_2, is calculated as:

$$u_2 = \sqrt{(\partial T_2/\partial v'_{50})^2 u_{50}{}^2 + (\partial T_2/\partial v'_{100})^2 u_{100}{}^2 + u_{c2}} \quad (24)$$

where

$$u_{c2} = 2 \cdot r_{50,100}(\partial T_2/\partial v_{50})(\partial T_2/\partial v_{100})u_{50}u_{100}$$

is the correlation component of u_2.

The relative T_1 uncertainty, u_{T1}, is then determined as a percentage of T_1, i.e.,

$$u_{T1} = 100 \cdot u_1/T_1$$

and the relative T_2 uncertainty, u_{T2}, is determined as a percentage of T_2, i.e.,

$$u_{T2} = 100 \cdot u_2/T_2$$

We will then determine the ratio of the relative uncertainty of each of the time parameters and the relative uncertainty of the voltage, for each of two cases. The results are listed in Table 2 and Table 3.

The relative voltage uncertainty u_v in Table 2 is expressed as a percentage of the voltage reading, and it is the same for all voltages. The relative voltage uncertainty in Table 3 is expressed as a percentage of the peak voltage v_{100}, considering that the absolute uncertainty is the same for all voltages in this case.

Table 2 Time parameter uncertainties and their ratios to the voltage uncertainty for Case 1.

Impulse Waveform	0.84/50	1.20/50	1.56/60	1.20/40
u_V (% of v)	0.4	0.4	0.4	0.4
u_{T1} (% of T_1)	0.563	0.564	0.565	0.564
u_{T1}/u_V	**1.41**	**1.41**	**1.41**	**1.41**
u_{T2} (% of T_2)	0.0	0.0	0.0	0.0
u_{T2}/u_V	**0.0**	**0.0**	**0.0**	**0.0**

Table 3 Time parameter uncertainties and their ratios to the voltage uncertainty for Case 2.

Impulse Waveform	0.84/50	1.20/50	1.56/60	1.20/40
u_V (% of v_{100})	0.4	0.4	0.4	0.4
u_{T1} (% of T_1)	1.053	1.045	1.046	1.040
u_{T1}/u_V	**2.63**	**2.61**	**2.62**	**2.60**
u_{T2} (% of T_2)	0.970	0.945	0.951	0.933
u_{T2}/u_V	**2.42**	**2.36**	**2.38**	**2.33**

The ratio u_{T1}/u_V in Tables 2 and 3 is dimensionless and can be used to approximate the sensitivity coefficient of the relative uncertainty of T_1 with respect to the relative voltage uncertainty. Similarly, u_{T2}/u_V can be used to approximate the sensitivity coefficient of the relative uncertainty of T_2. Tables 2 and 3 show that these sensitivity coefficients remain essentially constant when T_1 and T_2 are within the tolerance limits of the relevant IEC standard [1].

6 LIMITATION OF THE SENSITIVITY COEFFICIENTS

The sensitivity coefficients given in Table 1 are only valid when the errors of the defining voltages are small. To test the limit of the voltage errors for which the sensitivity coefficients are sufficiently accurate for practical application, comparisons were made between the calculated errors of T_1 and T_2 based on the sensitivity coefficients in Table 1 and those calculated for distorted waveforms.

A number of distorted waveforms were created from an ideal waveform with T_1 and T_2 values of 0.84 μs and 50.0 μs and non-linearity error curves such as the one in Figure 3. The T_1 and T_2 values of the distorted waveforms were then determined directly from the distorted waveforms, and their differences from the T_1 and T_2 values of ideal waveform were determined as the true errors of T_1 and T_2.

The T_1 and T_2 errors were also calculated using the sensitivity coefficients in Table 1 and the errors of the defining voltages from the distorted waveforms. The calculation was performed using the error propagation relationships:

$$\Delta T_1 = \Delta v_{30} \frac{\partial T_1}{\partial v'_{30}} + \Delta v_{90} \frac{\partial T_1}{\partial v'_{90}} + \Delta v_{100} \frac{\partial T_1}{\partial v'_{100}} \qquad (25)$$

$$\Delta T_2 = \Delta v_{50} \frac{\partial T_2}{\partial v'_{50}} + \Delta v_{100} \frac{\partial T_2}{\partial v'_{100}} \qquad (26)$$

where Δv_{30}, Δv_{90} and Δv_{50} are the errors of the defining voltages v_{30}, v_{90} and v_{50}, which are determined from the deformed waveforms.

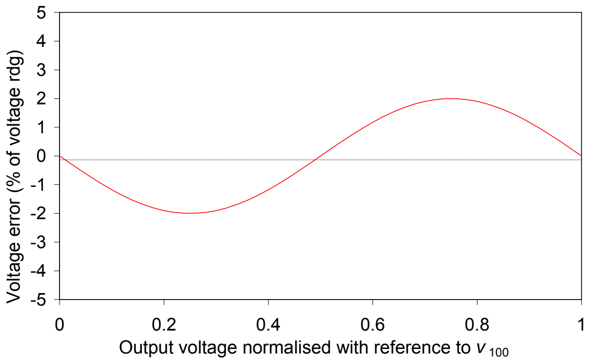

Figure 3 Hypothetical voltage non-linearity error curve used for creating a distorted impulse waveform.

Figure 4 shows the true T_1 errors determined directly from the distorted waveforms and the T_1 errors determined using the sensitivity coefficients in Table 1.

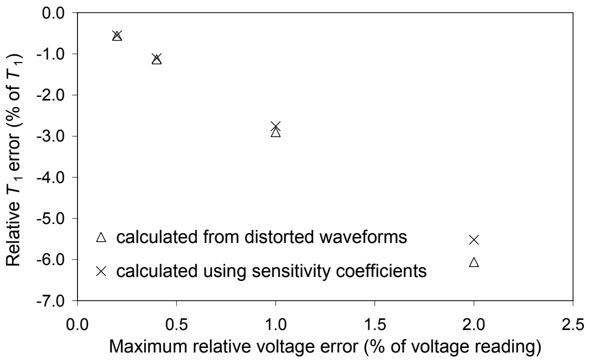

Figure 4 T_1 errors determined from deformed waveforms and calculated using the sensitivity coefficients.

The maximum relative voltage errors in Figure 4 were taken as the amplitudes of the voltage non-linearity curves (one of the curves is shown Figure 3, with an error amplitude of 2%). It can be seen that the T_1 errors calculated using the sensitivity coefficients is sufficiently similar to the true T_1 errors determined directly from the distorted waveforms, up to a voltage error of about 2%. Similar results were found for the effect on the calculated T_2 errors.

It follows that the conclusion for the T_1 and T_2 errors applies equally to the T_1 and T_2 uncertainties.

7 CONCLUSION

The sensitivity coefficients of the time parameters of the impulse voltage with respect to their defining voltages have been numerically determined. These sensitivity coefficients can be used for the determination of the time parameter uncertainties due to the uncertainties of the voltage measurement.

The sensitivity coefficient of the relative uncertainty of T_1 with respect to a uniform voltage uncertainty that is relative to individual voltage readings was found to be approximately 1.4, while that with respect to a uniform voltage uncertainty that is relative to the peak voltage was found to be approximately 2.6.

The sensitivity coefficient of the relative uncertainty of T_2 with respect to a uniform voltage uncertainty that is relative to individual voltage readings was found to be 0.0, while that with respect to a uniform voltage uncertainty that is relative to the peak voltage was found to be approximately 2.4.

For estimation of time parameter uncertainties in practical impulse voltage tests, these sensitivity coefficients can be considered valid for relative voltage uncertainties up to 2%.

8 REFERENCES

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