DIFFERENT TYPES OF WINDOWING TECHNIQUES FOR CONVOLUTION AND DECONVOLUTION IN HV TESTING SYSTEMS

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Abstract: Convolution and deconvolution data processing is often necessary and useful in testing and measuring systems. They are expressed with a simple multiple form of a matrix and a vector of two sets of digitized time series data. They are transformed into simple frequency spectra multiplication form with application of FFT to compute the process efficiently. In this step, various windowing techniques are applied to reduce digitized or circulation effects. It is an important issue, what type of window is suitable for the process. The appropriate windowing technique is dependent on a selected practical application case. The purpose of this paper is to present how to select the appropriate windowing technique for each practical case. First a numerical calculation example of a measured waveform considering impulse voltage divider characteristic is shown where no window is necessary. Secondly it is possible to estimate a transfer admittance waveform by deconvolving the response current waveform with the applied voltage where the exponential window is very useful. Lastly a computation example of frequency transfer functions (TF) of a power transformer in impulse tests is shown. The Nicolson and other mild windowing techniques are useful. Through these examples, the proposed utilization of the windowing techniques is validated.

1 INTRODUCTION

High voltage (HV) testing and measuring systems have been computerized with recent advances of digital recorders. This tendency has developed new digital signal processing techniques of high voltage measured data. Convolution and deconvolution operations are one of these and they are often necessary and/or useful in testing and measuring systems [1-7].

Convolution and deconvolution operations are expressed with a simple multiple form of a matrix and a vector of two sets of digitized time series data [7]. They are transformed into simple frequency spectra multiplication form with application of FFT, or the fast Fourier transform algorithm [8], to compute the process efficiently. In this step, various windowing techniques are applied to reduce digitized or circulation effects. It is an important issue, what type of window is suitable for the process? Or is any window unnecessary? If it is necessary, which window is appropriate for each practical case?

The appropriate windowing technique is dependent on a selected practical application case. The purpose of this paper is to present how to select the appropriate windowing technique for each case. First a basic numerical principle of convolution and deconvolution operations is described and it is explained why the FFT technique is utilized to process the operation efficiently. In this process numerical approximations or errors are inevitable and it is explained how to reduce them with application of windowing techniques for each case. Some practical cases with numerical examples are shown. First a numerical calculation example of a measured waveform considering its impulse voltage characteristic is shown. The measured waveform is expressed with convolution between the divider characteristic and the input voltage waveforms. In this case, no window is necessary. Secondly it is possible to estimate a transfer admittance waveform by deconvolving the response current waveform with the applied voltage. In this case, the exponential window is useful [9]. Lastly a computation example of frequency transfer functions (TF) of a power transformer in impulse tests is shown. The TF is calculated by deconvolving the response current with the applied input impulse voltage waveform in the frequency domain. The Nicolson and other mild windowing techniques are useful [10]. Through these examples, the proposed utilization of the windowing techniques is validated.

2 WINDOWING TECHNIQUES FOR CONVOLUTION AND DECONVOLUTION IN HV DATA PROCESSING

2.1 Convolution and deconvolution in HV data processing

Applying an voltage waveform $x(t)$ at an input winding terminal of a transformer, a response voltage or current waveform $y(t)$ of interest in Figure 1 is expressed with a transfer function $h(t)$.

$$y(t) = h(t) * x(t) = \int_0^t h(t-\tau)x(\tau)d\tau \qquad (1)$$

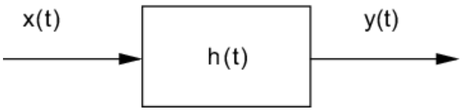
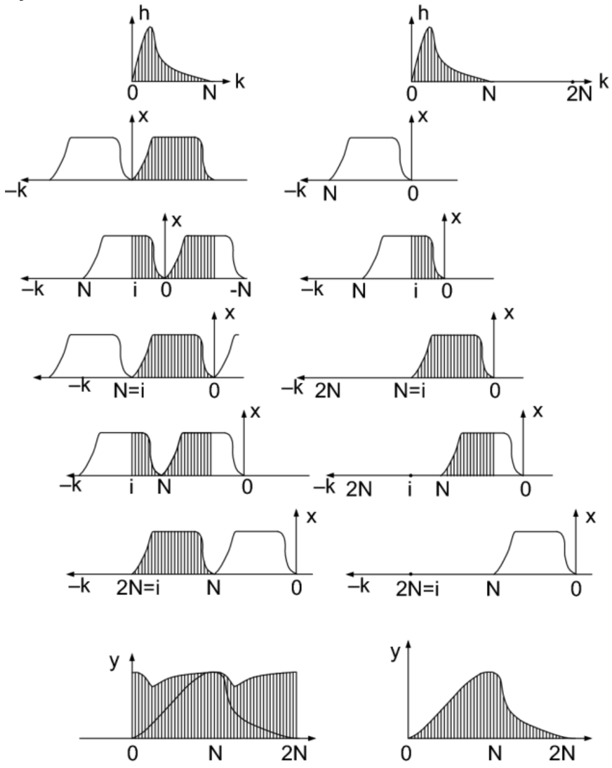


Figure 1: Relation of signals in HV measuring system.



(a) Without the zero-padding. (b) With the zero-padding

Figure 2: Time –domain convolution of x_i and h_i.

The above relation in the time domain is transformed into that in the frequency domain by the Fourier transform where the capital symbols mean data in the frequency domain.

$$Y(f)=H(f)X(f) \tag{2}$$

The transfer function is quotient of *Y*(*f*) and *X*(*f*).

$$H(f)=Y(f)/X(f) \tag{3}$$

2.2 Application of FFT

In a discretized high voltage system in Figure 2, an output signal y_i is given by convolution of an input signal x_i and the system transfer function h_i.

$$y_i=\Delta T\sum_{k=0}^{i-1}h_k x_{i-k} \tag{4}$$

where subscript *i* shows a sampling point at $t_i=(i-1)\Delta T$, ΔT is a time step size, and *N* is a sampling number.

A single shot waveform is processed accurately in convolution. However, it should be modified to be a repetitive waveform to apply the FFT algorithm. The repetitive waveform in Figure 2(a) is not processed accurately due to this circularization effect.

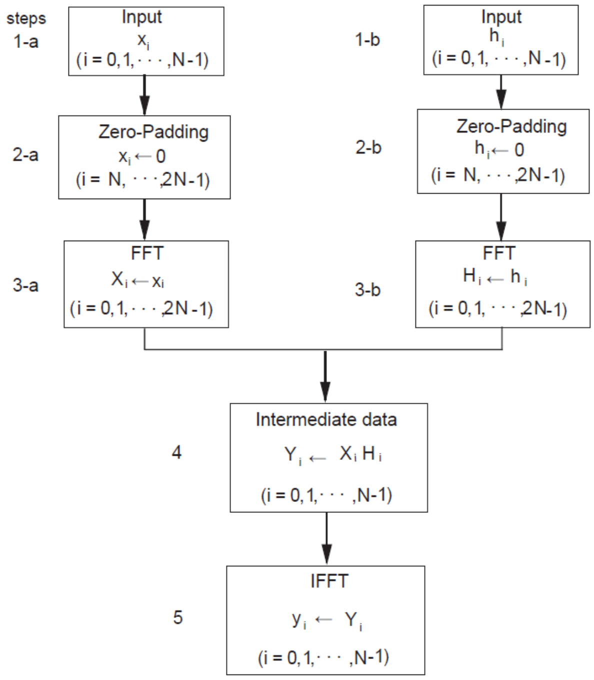


Figure 3: Numerical recipe for simple convolution in time-domain (No window is necessary).

$$\Delta T\sum_{k=0}^{N-1}h_k x_{i-k} \neq y_i \tag{5}$$

if $x_{i-N}=x_i\ (i=0,1,2,\cdots,N-1)$

One solution for this is padding of the same number *N* of zeros to the sampled data. After this zero-padding, the repetitive waveform in Figure 2(b) is processed accurately.

$$\Delta T\sum_{k=0}^{2N-1}h_k x_{i-k}=\Delta T\sum_{k=0}^{i}h_k x_{i-k}=y_i \tag{6}$$

if $x_{i-N}=0, h_{i+N}=0\left(i=0,1,2,\cdots,N-1\right)$

2.3 No window is necessary for simple convolution in time-domain

Zero-padded data are repetitive and processed efficiently in the frequency domain but the front half of the convolution result is identical with that of non-repetitive data of (4). This means that no window is necessary for simple convolution in time-domain.

The above numerical process is reduced to the recipe in Figure 3. *N* sampled input data for x_i, h_i are converted into intermediate data $X_i H_i$ by FFT after the zero-padding. Then multiplication $Y_i=X_i H_i$ of them is computed. Then *N* sampled convolved data in the time domain is accurately computed through the inverse FFT (IFFT).

The situation in deconvolution is quite different from that in convolution. Zero-padded data for x_i and y_i are given, and then they are solved for h_i. However the circularization effect cannot be eliminated even by the zero-padding.

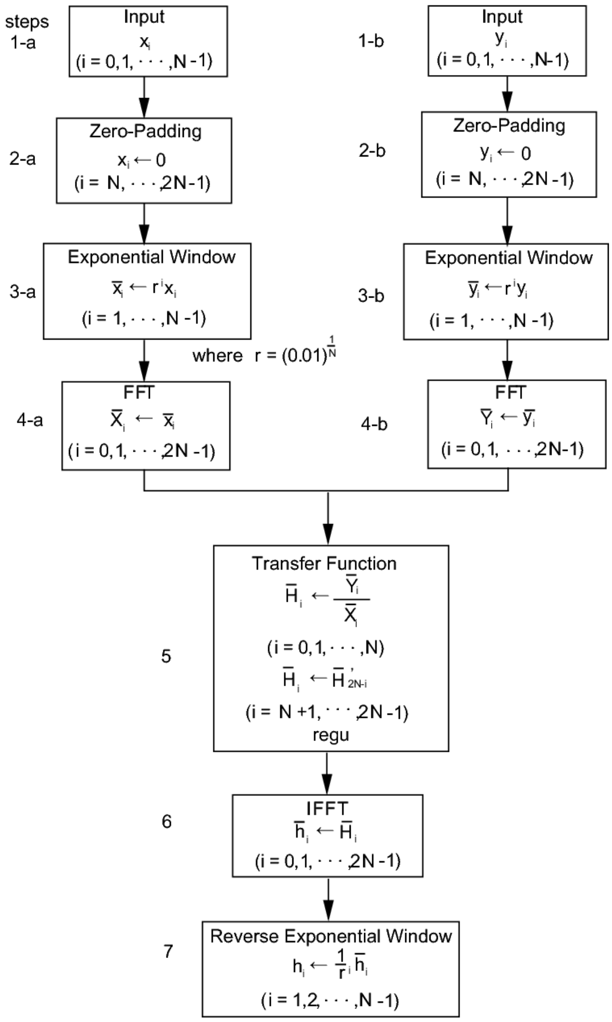


Figure 4: Numerical recipe for transfer function analysis in time-domain (Exponential window is useful).

2.4 Exponential window for time-domain deconvolution

One efficient way for reduction of the circularization effect is the exponential window method. Its basic idea lies in applying exponential decay weights on waveforms to reduce the truncation and circularization effects and in recovering the decay after convolution and deconvolution operations. In other words, discrete Fourier transforms are converted into discrete Laplace transforms. The optimum selection of decay constant α is dependent on waveforms, but empirically it is recommended it satisfies $\varepsilon^{-\Delta T(N-1)}=0.01$.

The above techniques are reduced to the recipe [7] of numerical processes in Figure 4. *N* sampled input data for x_i, y_i are converted into the frequency domain data through the zero-padding and the exponential windowing processes to reduce circularization and truncation errors. Then quotient of them is computed. Then *N* sampled transfer function data in the time domain is computed through the inverse FFT(IFFT) and reverse windowing processes.

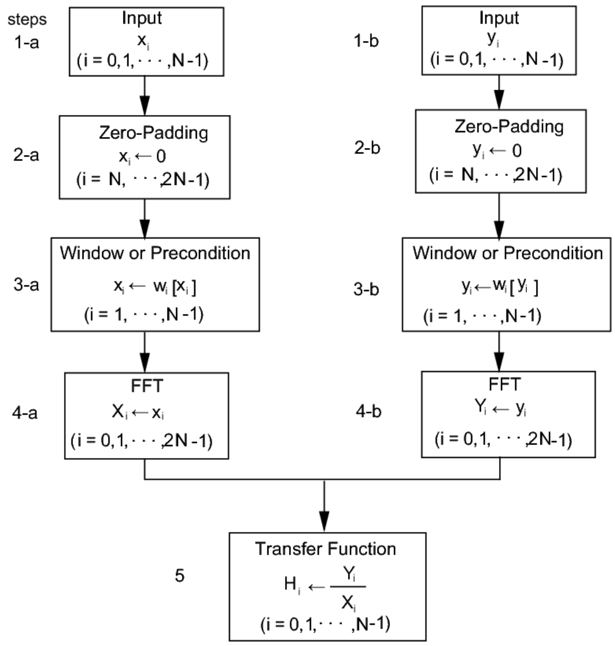


Figure 5: Numerical recipe for transfer function analysis in frequency-domain (Mild windows and Nicolson preconditioning techniques are useful).

2.5 Mild window or Nicolson preconditioning technique for frequency-domain deconvolution

When only frequency spectra data of the transfer function are necessary, they are computed according to the recipe in Figure 5. *N* sampled input data for x_i, y_i are converted into the frequency domain data X_i, Y_i through the zero-padding and the mild windowing or the Nicolson preconditioning technique [9] w_i[ ]. Then quotient H_i of them is computed. In this paper the Hanning window is applied only to the last 10% sampled data as the mild window technique.

2.6 Coherence function for noise estimation

It is necessary to estimate how much computed results are reliable for measured data. One technique for this estimation is a coherence function γ in sense of the reference [6]. The definition for *m* sets of measured data is the following.

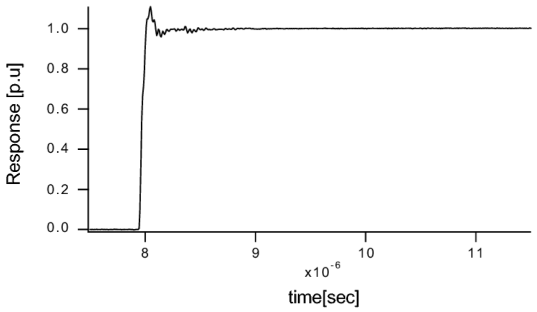
$$\gamma^2(f) = |S_{xy}(f)|^2 /(S_{xx}(f)S_{yy}(f)) \tag{7}$$

where

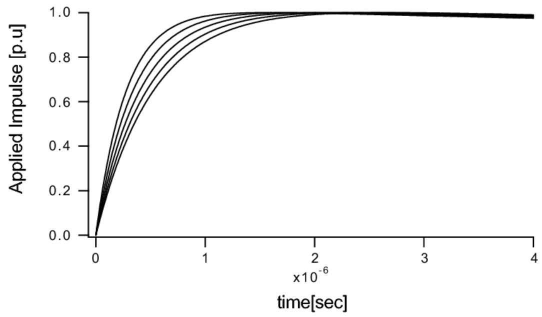
$$S_{xx}(f) = \Sigma\,|X(f)|^2 /m, S_{yy}(f) = \Sigma\,|Y(f)|^2 /m \tag{8}$$

$$S_{xy}(f) = |\Sigma X^*(f)Y(f)|/m \tag{9}$$

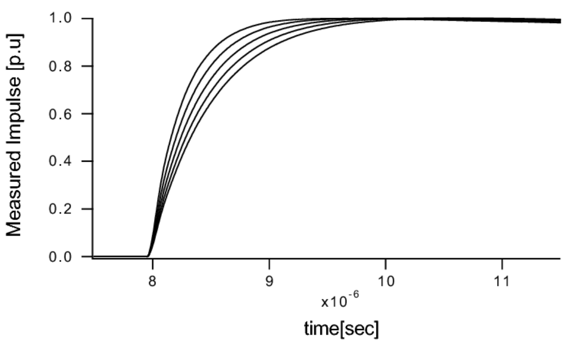
It means to estimate signal-to-noise (S/N) ratios by differences of computed results for *m* times.



(a) Step response of a divider



(b) Applied impulse



(a) Measured impulse

Figure 6: Error estimation of the impulse parameters measured with a divider.

3 NUMERICAL EXAMPLES

3.1 No windowing for time-domain convolution – divider response example

The output voltage waveform of a divider is computed from convolution between its unit step response and its input voltage waveform. The errors in the output waveform are estimated from the differences between the output and the input waveforms. The convolution method is a useful tool for this error estimation of the impulse parameters measured with a divider.

Figure 6 shows numerical example of this estimation. The unit step response of the divider is shown in Figure 6(a). Five standard lightning impulse waveforms of T_1=0.84, 1.02, 1.2, 1.38, 1.56 μs and T_2=60μs in Figure 6(b) are applied to the divider. The output impulse waveforms are computed according to the recipe in Figure 3 which are shown in Figure 6(c). For example, differences of the peak values are estimated less than 0.12%.

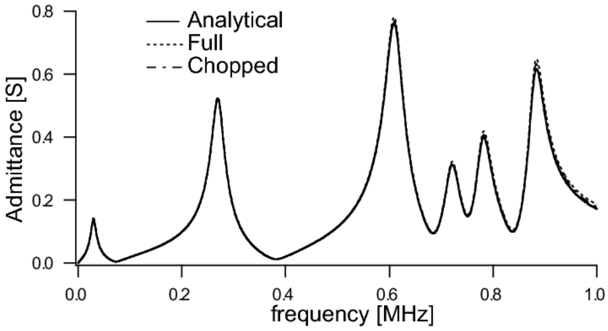
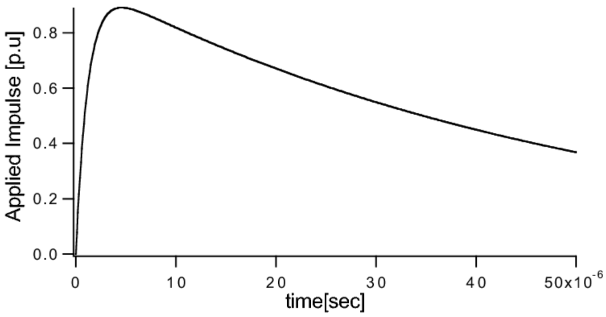


Figure 7: Frequency characteristic of a transfer admittance function.



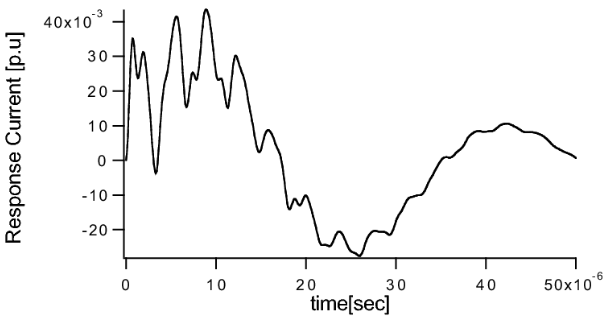


Figure 8: Applied full-voltage and response neutral current waveforms.

3.2 Exponential window for time-domain deconvolution – transfer function analysis example in time domain

Analytical data which are generated from a fractional polynomial model based on a transfer function characteristic are adopted as the second application example. Such analytical data have an advantage that precise analytical values can be computed.

Analytical frequency characteristic data in Figure 7 are synthesized by giving six sets of resonant frequencies, half-band widths, and peak values as in [2]. Then a full and a chopped impulse waveforms and their response neutral current waveforms are synthesized analytically. Full impulse case waveforms are shown in Figure 8.

Transfer functions from the analytical voltages and response currents are computed both for the full chopped cases according to the recipe in Figures 4 and 5, they coincided with the analytical solution in Figure 7.

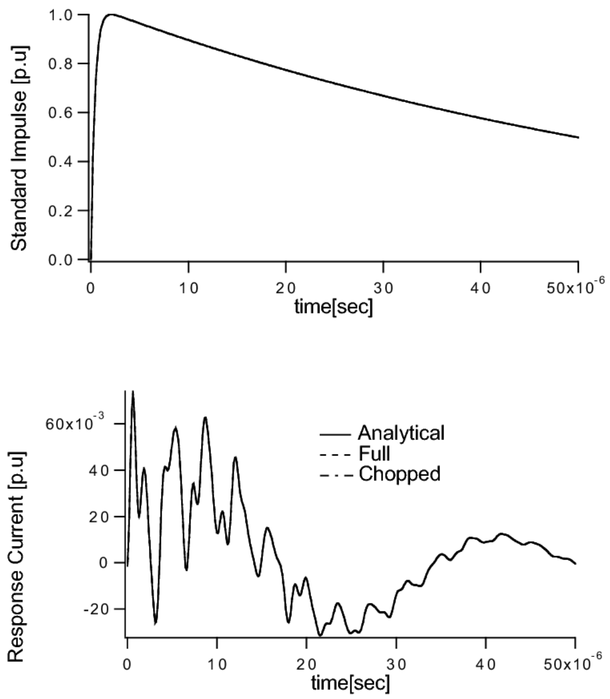


Figure 9: Standard full lightning impulse response.

Furthermore, standard full lightning impulse response waveforms are computed and compared between the full and chopped cases in Figure 9. They are coincided with the analytical solutions. This validates the proposed recipes and the developed numerical processing methods.

3.3 Mild Hanning window example – transfer function analysis in frequency domain

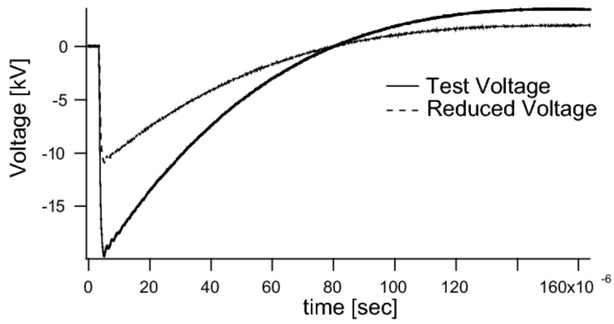
Measured waveforms in an actual transformer are adopted as the last application example. A reduced and a full test impulse voltage waveforms in Figure 10(a) are applied and their response currents in Figure 10(b) are measured. Computed transfer functions from them are shown in Fig.10(c) according to the recipe in Figure 5. They are almost coincided.

A coherence function from these two sets (m=2) is computed and shown in the same figure. From the coherence function, the computed transfer functions are effective about up to 2MHz.

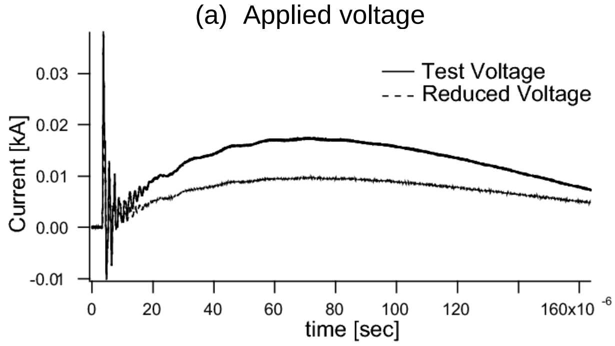
4 CONCLUSIONS

This paper has proposed how to utilize the windowing techniques for convolution and deconvolution data processing which is often necessary and useful in HV testing and measuring systems.

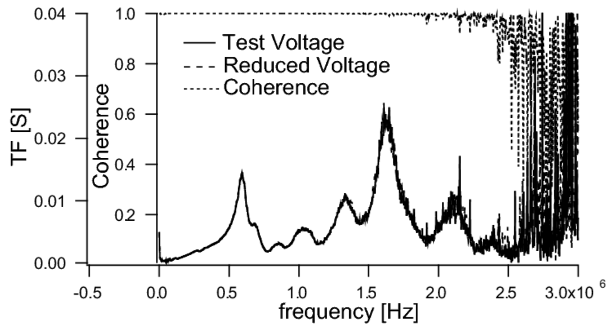
The appropriate windowing techniques are dependent on practical application cases. They are categorized into the following three types.



(a) Applied voltage



(b) Response current



(c) Transfer and coherence functions

Figure 10: Coherence and transfer admittance functions of measured waveforms.

(1) No window is necessary and zero-padding is enough for simple convolution in time-domain. Numerical response computation cases, for example, output waveforms considering an impulse voltage divider characteristic belong to this type.

(2) When the final results after deconvolution operation are waveforms in the time domain, the exponential window is useful to reduce circularization and truncation errors. Most cases of transfer function processing in the time domain belong to this type.

(3) When the final results after deconvolution operation are characteristics or spectra in the frequency domain, some mild window and the Nicolson preconditioning techniques are useful. Computation cases of frequency transfer functions including impulse tests of power transformers belong to this type.

Through these three types of application examples, the proposed selection and utilization of the windowing techniques is validated.

5 ACKNOWLEDGMENTS

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