# EXPONENTIAL DISTRIBUTION OF THE CLOUD-TO-GROUND LIGHTNING FLASH DENSITY: LLS RESULTS IN CHINA'S THREE GORGES AREA IN A DECADE, 2000~2009

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**Abstract**: The Mean value of Lightning flash Density is used as a criterion to assess the Lightning Stroke Risk, e.g. transmission line and building. Theoretically, the mean value approximates to the expectation value of lightning flash densities in a certain region, as long as the record time of the lightning data is long enough. We analyze the density characteristics of 1.27 million Cloud-to-Ground (CG) lightning flashes recorded by Lightning Locating System (LLS) in China's Three Gorges Area (110°-112.5°E, 30°-32° N) during a decade (2000-2009). Exponential distribution is assumed to describe the frequency distribution of lightning flash densities in the decade. Single-grid random sample and expanded-grid sample are used to validate the hypothesis. And the mean and the standard deviation (STDEV) of lightning flash densities are also compared in the whole area (because Mean and STDEV of exponential distribution are equal). We propose to take the bigger one in Mean and STDEV as the expectation of lightning flash densities. It provides a better correlation with historical record of lightning flash over point than the mean lightning flash density, on 220kV Ge-Chen (from Gezhouba to Chenjiachong transformer substation) electric transmission line in this decade.

# **1** INTRODUCTION

The lightning flash density is a very sensitive parameter in High Voltage Transmission Line Lightning Protection Project. Statistics show that lightning flashover accidents of the transmission line are proportional to Lightning density<sup>[1]</sup>. As lightning activity is highly random, the average density during many years is used to measure the lightning characteristics in a given region <sup>[2-3]</sup>. However, the fluctuation with time of lightning density is not defined by these parameters, thus it is necessary to estimate the probability of lightning density which is greater than the mean value. The statistical data of lightning locating system (LLS) [4-5] in the Three Gorges area in Western Hubei during 2000-2009 are analyzed. We assume that lightning flash density obey the exponential distribution, and propose that the bigger one between the mean and STDEV can be used for estimating lightning density expectations in the engineering.

# 2 LIGHTNING LOCATING SYSTEM DATA

The lightning locating system in Hubei Province, China, is founded by Hubei Electric Power Company, and formed by IMPACT (DF & TOA) detection stations group. Up to now, the detection and localization scope of the LLS have covered all power grid in Hubei, whose theory positioning accuracy can be less than or Equal to 1km, in which the detection efficient approximates 90%. A decade operation experience shows the detection results approach the theoretical analysis value. The research scope of LLS data mainly covers the Three Gorges Hydropower Station and its perimeter zone in western Hubei, between  $110^{\circ}$ -112.5°E and  $30^{\circ}$ -32°N, and the time span cross a decade from 2000 to 2009. The data chief record the first return stroke of CG lightning flash, which hazards to the transmission lines the most. The volumes of the data given by the LLS of the Three Gorges area in western Hubei in 10 years are as shown in Figure 1, and the annual average lightning flash record is about  $10^{5}$ .



**Figure 1:** Lightning locating system data in Three Gorges Area in Western Hubei from 2000 to 2009

#### 3 ANALYSIS OF LLS DATA IN 2000-2009

# 3.1 Grid statistical methods of CG lightning flash density

Divide the whole study area of Three Gorges area in western Hubei into 0.01°x0.01° grids (the size o f every grid is about 1km×1km and the amount of grids is 200×250. The frequency of CG lightning in a single grid each year is CG lightning flash density (fl/km<sup>2</sup>/yr).

The grid CG lightning flash density of Three Gorges area in western Hubei in 2000-2009 is calculated and the correlation coefficient within lightning flash density and MEAN of Corresponding grid in 10 years form a Correlation Matrix in Table 1. The correlation coefficient of any two years being less than 0.3, which shows the great randomness of CG lightning flash density and that between MEAN and each year increasing to 0.5, shows the MEAN has synthesized the principal character of CG lightning flash density in 10 years. However the CG lightning flash density of each year does not equate the MEAN, so that the distribution characteristic requires further study.



**Figure 2:** Lightning flash density and its MEAN from 2000 to 2009

Table 1	: Correlation	matrix of I	lightning	flash in	Three Gorges	Area in	Western Hubei
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	2000	2001	2002	2003	2004	2005	2006	2007	2008	2000	
	2000	2001	2002	2003	2004	2005	2006	2007	2006	2009	
2000	1.000	0.106	0.163	0.052	0.123	0.209	0.181	0.212	0.134	0.226	0.491
2001	0.106	1.000	0.093	0.097	0.135	0.128	0.134	0.127	0.072	0.082	0.343
2002	0.163	0.093	1.000	0.078	0.047	0.141	0.148	0.123	0.093	0.131	0.384
2003	0.052	0.097	0.078	1.000	0.276	0.097	0.117	0.176	0.124	0.093	0.485
2004	0.123	0.135	0.047	0.276	1.000	0.137	0.133	0.206	0.147	0.091	0.536
2005	0.209	0.128	0.141	0.097	0.137	1.000	0.220	0.189	0.140	0.123	0.482
2006	0.181	0.134	0.148	0.117	0.133	0.220	1.000	0.211	0.115	0.193	0.508
2007	0.212	0.127	0.123	0.176	0.206	0.189	0.211	1.000	0.150	0.191	0.551
2008	0.134	0.072	0.093	0.124	0.147	0.140	0.115	0.150	1.000	0.161	0.473
2009	0.226	0.082	0.131	0.093	0.091	0.123	0.193	0.191	0.161	1.000	0.477
MEAN	0.491	0.343	0.384	0.485	0.536	0.482	0.508	0.551	0.473	0.477	1.000

#### 3.2 Lightning flash density distribution of Single grid samples

Two grid samples are random taken in research area, whose histogram of CG lightning flash density is shown in Figure 3 left area. Exponential function is adopted to fit the distribution of lightning flash density, as shown in the right area of Figure 3.

Exponential function fitting formula is

$$P_{N_g} = a\lambda e^{-\lambda N_g} \tag{1}$$

 $N_{g}$  is CG lightning flash density;  $\lambda$  is rate parameter; *a* is amplitude coefficient.

So the maximum likelihood estimating of exponential distribution is

$$\hat{\lambda} = \frac{1}{\overline{N_g}} \tag{2}$$

When  $\lambda$  is determined, the amplitude coefficient *a* can be reached by Least Square Fit.



**Figure 3:** Histograms and fitting curves of 2 geographic grid samples in Three Gorges Area in Western Hubei

**Table 2:** Comparison of fitting parameters of 2 grid samples

Fitting parameters	â	а	R-Square
Grid sample 1	1.43	9.42	0.91
Grid sample 2	0.256	12.34	-0.79

Fitting parameters is shown in table 2, in which the closer R-Square parameters to 1, the better the fitting result is. Hence the fitting effect of Grid Sample 1 is better. The time series of 10 years is small, when the distribution of CG lightning flash density is non-uniform, fitting is difficult, and which will affect the MEAN and Expectation greatly as a result of lacking of samples. So the following part will testify the CG lightning flash density exponential distribution by using adjacent grids sample expansion.

#### 3.3 Lightning flash density analysis of expanded grid samples

Since the observation time of LLS in Three Gorges area in western Hubei is limited, there are only 10 years CG lightning flash densities in a same area. In order to expand samples, we replenish it using the CG lightning flash density data of adjacent grids. In this case, the 3×3 grid is researched; the proportion of research area being 3km×3km; the distribution of lightning flash in such small geographic scales is basically the same; but small difference of CG lightning flash density value between grids is kept when selecting samples. The data samples come to 90 from 10 after expanding, which has increased 9 times. The statistic curve after expanding is as shown in Figure 4.



**Figure 4:** Histograms and fitting curves of 2 extended grid samples in Three Gorges Area in Western Hubei

As shown in Figure 4, through sample expansion, two R-Square parameters exceed 0.6. Which means that, as samples adds, the law of the exponential distribution of the CG lightning flash density in Three Gorges area in western Hubei is clearer.

# 3.4 Overall statistics

The density function of exponential distribution is

$$f(x;\lambda) = \begin{cases} \lambda e^{-\lambda x}, x \ge 0\\ 0, x < 0 \end{cases}, \lambda > 0$$
(3)

$$E(x) = \frac{1}{\lambda} = \sqrt{D(x)}$$
(4)

This indicates that the MEAN equates STDEV of exponential distribution.

From the overall data (50000 Grids), the MEAN and STDEV of CG lightning flash density in every grid is calculated, as shown in the figure 5. The slope of linear fit in the figure is 0.93 (meaning the MEAN of lightning flash density approximately equates its STDEV), suggesting that CG lightning flash density obeys exponential distribution.



**Figure 5:** Linear relation between mean and STD of CG lightning density in Three Gorges Area in Western Hubei in a decade

#### 4 LIGHTNING FLASH DENSITY ANALYSIS OF 220KV GE-CHEN TRANSMISSION LINE

Reviewing the exponential fit of the CG lightning flash density distribution, we propose to take the bigger one between Mean and STDEV, as the expectation of lightning flash densities. It provides a more rigorous criterion of Lightning Stroke Risk Assessment than the mean lightning density criterion.

Taking the typical 220kV Ge-Chen transmission line in three gorges area for example, the fulllength of Ge-Chen line is 33.37 km, and it starts at Gezhouba, ends at Chenjiachong substation (nearing the three gorges dam), officially put into operation in December 1994. Ge-Chen line includes 79 base towers, with full double overhead grounding wire. The type of overhead grounding wire is GJ-50 and OPGW. Average lightning flash density of Ge-Chen line is 4.7 fl/km<sup>2</sup>/yr; The maximum value is 6 fl/km<sup>2</sup>/yr and the minimum is 3 fl/km<sup>2</sup>/yr. Figure 6 shows the MEAN and Expectation (the larger value between MEAN and STDEV) of lightning flash density of lines corridor, and compares the difference of them.



**Figure 6:** Comparison of MEAN and Expectation of lightning flash density and flashover point on 220kV Ge-Chen transmission line

Figure 6 also shows 14 historical lightning flashover points of Ge-Chen transmission line from 2000 to 2009, corresponding with MEAN and Expectation of lightning flash density. Among them, the MEAN of 9 lightning flashover points are upon or equal to 5 fl/km<sup>2</sup>/yr, which takes 62.3% of the total; The Expectation of 12 lightning flashover points are upon or equal to 5 fl/km<sup>2</sup>/yr, which takes 85.7%. It implies the Expectation of lightning flash density has better correlation with the lightning flashover of transmission lines.

#### 5 CONCLUSION

1) The exponential distribution is assumed to describe the frequency characteristic of CG lightning flash density of Three Gorges area in western Hubei from 2000 to 2009, according to the statistics on lightning locating system data of Hubei power grid.

2) The exponential distribution shows that, the probability of the repeated high CG lightning flash density is very low in the same area, and the most common lightning flash density is below the MEAN.

3) In order to compensate the limited samples of lightning flash density of time series, we propose to take the bigger one in Mean and STDEV as the expectation of lightning flash densities in Transmission Line Lightning Protection Project to assess the Lightning Trip-out Risk of transmission line more exactly.

4) The Expectation of lightning flash densities provides a better correlation with historical record of lightning flashover point than the MEAN on 220kV Ge-Chen electric transmission line between 2000 and 2009.

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