

# USAGE OF STATISTICAL MODELING OF LIGHTNING LEADER ADVANCEMENT PROCESS IN THE LAST STROKE PHASE FOR DETERMINATION OF LIGHTNING PROTECTION SYSTEM PARAMETERS

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**Abstract:** In the paper, the experimental data on the electrical physical characteristics of long spark gaps and lightning are used to create a lightning leader development model in the last stroke phase. The model is verified by comparison of the calculated probabilities of lightning attachments to air terminals and protected objects with normalized level. In order to determine the influence of ground system resistance on protection ability for lightning air terminals, lightning attachments with different ground system resistances have been modeled. The proposed method has been implemented for the calculation of lightning strokes probability to objects of an extended facility with oil storage tanks.

## 1 INTRODUCTION

At present, lightning protection zones provided by lightning air terminals (LAT) are specified by normative documents [1-3]. However, available methods to determine these lightning protection zones do not consider all of the complex phenomena necessary to define the complete analysis. Lightning damage to mission critical objects such as missile launch complexes or oil storage tanks may lead to significant material losses and even catastrophes. It becomes imperative to model the lightning leader advancement process allowing more exact estimation on lightning damage probability. Such models are also required to determine the efficiency of new lightning protection technology.

In some other studies [4-6], electrostatic field distribution during thunderstorm conditions were calculated near LAT and the protected objects. In [7], a 3D stochastic fractal model of lightning propagation was proposed. However, this model does not take into account all of the important parameters such as lightning leader propagation velocity, leader potential, and external electric field strength. In [8], the so-called electro geometric model was used. A drawback of this approach is the disregard of the influence of the electric field to determine velocity and direction of lightning leader movement. In [9-11], the fractal models describing propagation of stepped leaders toward the ground and development of upward leaders from the grounded objects were presented. However, these models are not intended to determine the reliability of lightning protection.

In order to overcome the limitation on the study of lightning protection zones, an elaborate stochastic

mathematical model describing the development of lightning leader in the final stage of its propagation to ground needs to be analyzed with consideration of complex parameters. A large number of experimental data on the electrical physical characteristics of long spark gaps and lightning, as well as on the protected zones of different lightning air terminals have been considered to develop the elaborated model.

## 2 ASSUMPTIONS OF THE MODEL

The model described in the reference [12] is based on generalization of the experimental investigations of high voltage impulse breakdown of "rod-plane" long gaps and lightning [13]. The main assumptions of the model are follows.

The last stage of a lightning leader channel movement to grounded objects begins when the streamer zone of descending lightning leader channel reaches them. It is supposed that the "last stroke" is a process of leader channel movement through the streamer zone.

It is supposed that velocity and acceleration of lightning leaders depend on the lightning potential and on the angle between vectors of the lightning leader's velocity and electric field strength.

There are two conditions to consider, the fulfilment of one of which means that a corresponding grounded area may be struck by lightning in a considered numerical experiment. This first condition is a decrease of specific resistance ( $\rho_F$ ) of a successful streamer channel to the level close to leader's resistance:

$$\rho_F = k_F \cdot \rho_L \quad (1)$$

where  $\rho_L = 10 \Omega/\text{m}$  is resistance per leader channel length; and

$$k_F = 1-10 \quad (2)$$

The second condition supposes that one of the competing spark channels moving from a lightning leader in its streamer zone reaches a considered grounded area.

The equation for  $i_F$ , i.e., current flowing through a grounded object or an earth section to which a streamer has been connected after beginning of the last stroke phase may be written as:

$$i_F = U_L / (\rho_F \cdot L_{st} + R_R + R_G) \quad (3)$$

where  $U_L$  is lightning potential;  $\rho_F$  is resistance per streamer's channel length in the last stroke phase;  $R_R$  is resistance of the over-ground part of an object,  $L_{st}$  is decreasing length of a streamer approaching the earth, and  $R_G$  is ground system resistance of an object.

It is supposed that at the rising of the discharge current through a streamer and its channel widening, the streamer's resistance reduces by the

empirical law [14]:  $\rho_F(i_F, t) = 1 / [k_b \cdot \int_0^t i_F^{2/3} dt]$  (where

$k_b = (4\pi\sigma^2/\rho_0\xi)^{1/3}$ ;  $\rho_0$  is density of gas in which discharge takes place;  $\sigma$  is specific conductivity of the discharge channel;  $\xi=4.5$  is a coefficient).

The principle analogous to "Least Time – Maximum Probability" [15] is used to describe a lightning leader "selection" process of a place that is being struck. It is considered that the probability of lightning leader attachment to a grounded area is inversely proportional to the time of its reaching.

All the possibilities of lightning strokes origination from different nodes of the area above an investigated domain are considered. The model supposes calculation of the spark's advancement time to each zone of an area under evaluation. The algorithm takes into account all the possible range of lightning leader channel potentials with corresponding probabilities of their appearance [2, 16].

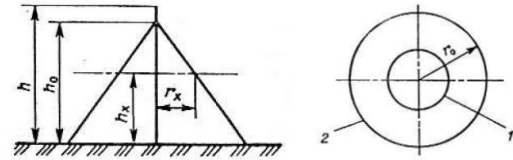
It is assumed that the lightning leader will strike only those places for which expected time to reach does not exceed  $1.1 \times t_{\min}$ , where  $t_{\min}$  is the minimum time of lightning leader propagation to a grounded spot in a considered numerical experiment. This 10 % dispersion corresponds to the values of probable discharge times and

voltages of multi-meter air gaps with a sharply non-uniform electric field [13].

For further calculations, the relationship between lightning leader potential ( $U_L$ ) and lightning current ( $I_L$ ) was used which was given by  $U_L = Z_L \times I_L$  (where  $Z_L \sim 800-1200 \Omega$ ) [13]. It gives more pessimistic results on the prognosticated number of lightning strokes than usage of another known relationship [2, 16]  $U_L = 9.4 \cdot I_L^{2/3}$  (where  $U_L$  [MV],  $I_L$  [kA]).

### 3 VERIFICATION OF THE MODEL AND DETERMINING INFLUENCE OF GROUNDING CONDITIONS ON LIGHTNING AIR TERMINAL PROTECTION ABILITY

To verify the proposed model, probabilities of lightning strokes to objects located in the normalised zone [1] that provide protection with some extent of reliability have been calculated. The protection zone of a lightning air terminal with the height  $h$  is shown in Figure 1.



**Figure 1:** Normalised protection zone of a lightning air terminal [1]

In order to protect the structure with 95 % reliability, the following parameters of the zone are specified [1]:  $h_0 = 0.92h$ ,  $r_0 = 1.5h$  and  $r_x = 1.5(h - h_x)/0.92$ . For the assigned object's parameters  $h_x$  and  $r_x$ , the height of the appropriate lightning air terminal may be defined as follows:  $h = (r_x + 1.63h_x)/1.5$ .

The results of calculated lightning attachment probability for a lightning air terminal with height  $h$  ( $h=12\text{m}$ ), a protected rod with height  $h_x$  ( $h_x=6.5\text{m}$ ), being separated from the air terminal by  $r_x$  ( $r_x=7\text{m}$ ), are presented in Table 1 ( $N=1$ ). Calculations are performed at annual average flash density per square kilometre  $N_m=1$ . As observed from these data, the coefficient  $K_1$  value ( $K_1=0.0194$ ) is less than 0.05, so the rod is protected with no less than 95 % reliability as claimed for the protection zone shown in Figure 1 [1]. These data are supported by physical modelling and field observations. If the height of the rod is increased up to  $h_x=9\text{m}$ , it will not match the protective zone and predicted number of lightning strokes exceeds 5 % (see Table 1,  $N=2$ :  $K_1=0.13 > 0.05$ ).

The calculation of the lightning attachment probability was also performed for the system consisting of a parallelepiped with dimension of  $8 \times 8 \text{ m}^2$  and the height  $h_x=6.5\text{m}$ , which is located

within the protected zone of a lightning air terminal. The LAT is installed on the parallelepiped upper base with total height equal to  $h=12$  m. The results are shown in Table 1 ( $N=3$ ). It can be observed that such an object is protected with not less than 95 % reliability, as the coefficients values:  $K_1=0.0038$  and  $K_2=0.02$  are less than 0.05.

**Table 1:** Forecasted number of lightning strokes per year to a lightning air terminal ( $N=1$ , 2) and a parallelepiped ( $N=3$ )

N	$P_{L1} \times 10^2$	$P_2 \times 10^4$	$P_{ar} \times 10^4$	$K_1 \times 10^2$	$K_2 \times 10^2$
1	0.185	0.359	0.359	1.94	1.94
2	0.172	2.26	2.26	13.1	13.1
3	0.182	0.372	0.0706	0.388	2

In Table 1,  $P_{L1}$  is the forecasted number of lightning strokes to the lightning air terminal;  $P_2$  is the forecasted number of lightning strokes to the protected rod ( $N=1$ , 2) or total forecasted number of strokes to the object – parallelepiped ( $N=3$ );  $P_{ar}$  is the maximum level of forecasted number of lightning strokes in the territory protected by the lightning air terminal;  $K_1=P_{ar}/P_{L1}$ ;  $K_2=P_2/P_{L1}$ .

The forecasted numbers of lightning strokes are calculated without considering the effect of ground system resistance of lightning air terminals and protected objects. In order to evaluate the possible influence of a lightning air terminal's grounding conditions, lightning stroke probability distribution are calculated for "lightning air terminal – rod" (Table 2,  $N=1$ ) and "lightning air terminal - object" (Table 2,  $N=2$ ) systems with variable ground resistance. The results of calculation are shown in Table 2. The geometry of system 1 and system 2 are same as in Table 1,  $N=1,3$ : the objects located in the protection zone. The ground system resistance of the lightning air terminal is defined as  $R_{L1}$  while ground system resistance of the protected rod or parallelepiped object is considered as  $R_{L2}$ . It is supposed that ground system resistance of the protected objects is much smaller than  $R_{L1}$  ( $R_{L2} \ll R_{L1}$ ) and the value of  $R_{L1}$  is varied for calculation.

**Table 2:** Forecasted number of lightning strokes per year at different lightning air terminal grounding resistance

$R_{L1}$						
$10^4 \Omega$		$10^5 \Omega$		$10^6 \Omega$		
N	$P_{L1} \times 10^2$	$P_2 \times 10^4$	$P_{L1} \times 10^2$	$P_2 \times 10^4$	$P_{L1} \times 10^2$	$P_2 \times 10^4$
1	0.184	0.376	0.131	2.37	0.093	4.79
2	0.18	0.397	0.0904	3.51	0.0476	6.14

As observed from Table 1 and Table 2, ground system resistance less than  $10^4 \Omega$  does not

influence lightning attachment probability of the object for the considered cases. When resistance of lightning air terminal grounding is about  $0.1 - 1 \text{ M}\Omega$ , the forecasted number of lightning strokes to the protected object may increase up to 12 times for the case with two rods ( $N=1$ ) and up to 15 times for the case with parallelepiped object ( $N=2$ ). Moreover, for variant 2, the ratio  $P_{L1}/P_2$  may change from 45 for lightning air terminal grounding resistance ( $R_{L1}$ ) less than  $10^4 \Omega$  to 0.8 for  $R_{L1}$  equals to  $10^6 \Omega$ .

In previous calculations, the coefficient  $k_F$  in the formula (1) is assigned a value equal to 10. Numerical analyses have shown that variation of  $k_F$  within a limit of 1 - 10 does not change lightning attachment probability when grounding resistances of objects and air terminals are not accounted for. This variation is minor when an object is located in the protected zone of an air terminal as shown in Table 2. However, when an object is taller than the protected zone upper boundary, value of  $k_F$  influences the results of lightning stroke probability calculation. The calculations have shown that the larger the value of  $k_F$ , the more optimistic is the result on the number of lightning attachments to objects.

Let us consider the worst case scenario when value of  $k_F$  equals to 1. The numerical analyses carried at the levels of potential,  $U_L=28 \text{ MV}$ , and current,  $I_L=30 \text{ kA}$ , (probability of such lightning appearance equals to 20 % [2,16]) have shown that for tall objects which are located slightly above the protected zone boundary, the critical level of ground system resistance influencing the probability of lightning attachment may reduce to  $100 \Omega$  (see Table 3). The calculations are performed for the "lightning air terminal - object" system where the object is a parallelepiped with dimensions  $2 \times 2 \text{ m}^2$  and height  $h_x=10.5 \text{ m}$ . The lightning air terminal is installed on its upper base having total height of 12m.

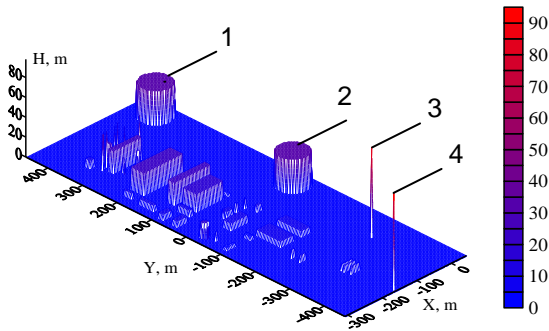
**Table 3:** Calculated coefficient  $K_2=P_2/P_{L1}$  - ratio between the forecasted number of lightning strokes to the object (not fully located within the protection zone) and the air terminal at different grounding resistances

$R_{L1}$					
0 $\Omega$	100 $\Omega$	200 $\Omega$	400 $\Omega$	$10^3 \Omega$	$10^4 \Omega$
0.018	0.024	0.03	0.034	0.039	all strokes to the object

#### 4 EXAMPLES OF THE MODEL USAGE

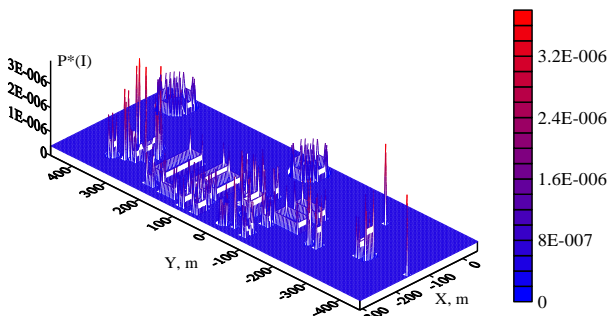
A detailed example of the model usage is illustrated considering the lightning stroke probability to an extended facility with oil storage tanks. A plan of the facility with heights ( $H$ ) and

locations (X,Y) of all associated objects is presented in Figure 2. Figure 3 to Figure 10 illustrate calculated distributions of coefficients  $P^*$  and  $P_o^*$  proportional to the probability of lightning strokes, defined as  $P^*=P_k/S_\Sigma$  (where  $P_k$  is probable frequency of lightning strokes to k-th node) and  $P_o^*=P_o/S_\Sigma$  (where  $P_o$  is the predicted number of all lightning strokes to n-th object of the considered facility) for every object. Here  $S_\Sigma$  is the area of an investigated facility which equals to  $S_\Sigma=150 \times 120 \text{ m}^2$  for the illustrated example (Figure 2). The calculations are performed at  $N_m=1$ .

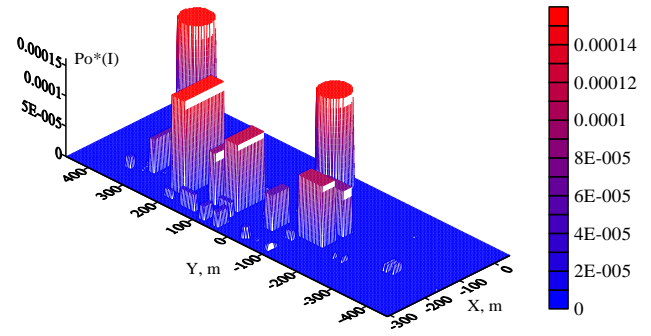


**Figure 2:** Plan of the investigated object; 1 and 2 are oil storage tanks; 3 and 4 are lightning air terminals

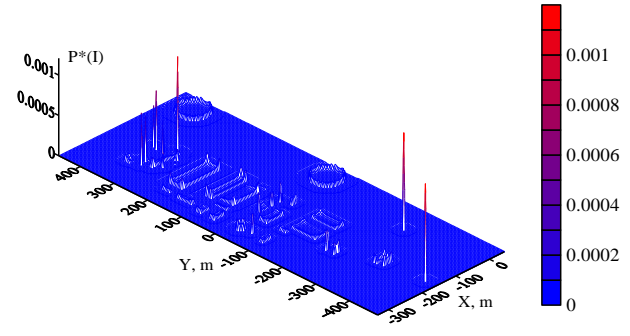
Figure 3 and Figure 4 present the probability of lightning strokes with  $U_L=-8 \text{ MV}$  ( $P_{\Delta I}=0.01$  [2, 16] - probability of appearance of lightning with such level of current and corresponding potential); Figure 5 and Figure 6 represent probabilities of lightning strokes with  $U_L=-28 \text{ MV}$  ( $P_{\Delta I}=0.2$ ); and Figure 7 and Figure 8 show probabilities of lightning strokes with  $U_L=-100 \text{ MV}$  ( $P_{\Delta I}=0.02$ ). Distributions of the calculated coefficient  $P^*$  for the full range of probable lightning stroke potentials (from  $-8 \text{ MV}$  to  $-100 \text{ MV}$ ) are shown in Figure 9 and Figure 10. Analysis of the plots in Figure 3 to Figure 10 shows that probability of lightning attachment is strongly dependent on the value of the lightning potential level.



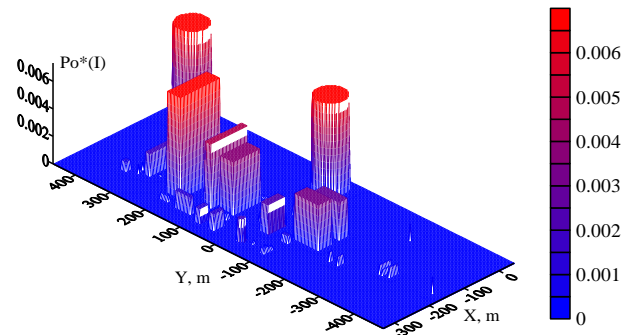
**Figure 3:** Distribution of the coefficient  $P^*$  at  $U_L=-8 \text{ MV}$  ( $P_{\Delta I}=0.01$ )



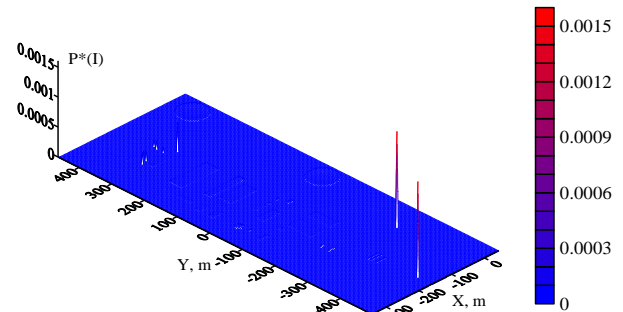
**Figure 4:** Distribution of the coefficient  $P_o^*$  at  $U_L=-8 \text{ MV}$  ( $P_{\Delta I}=0.01$ )



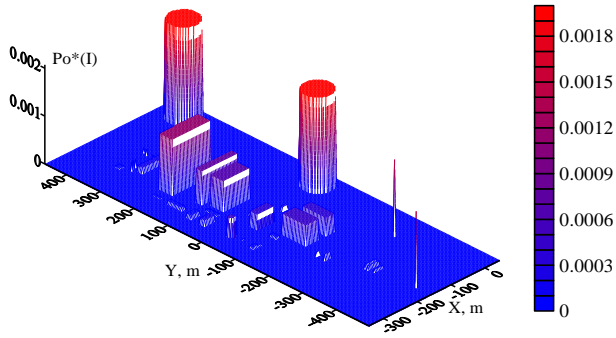
**Figure 5:** Distribution of the coefficient  $P^*$  at  $U_L=-28 \text{ MV}$  ( $P_{\Delta I}=0.2$ )



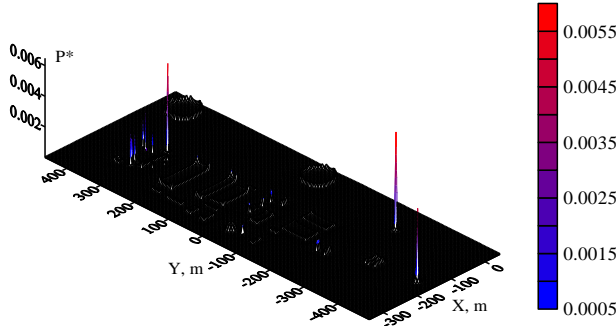
**Figure 6:** Distribution of the coefficient  $P_o^*$  at  $U_L=-28 \text{ MV}$  ( $P_{\Delta I}=0.2$ )



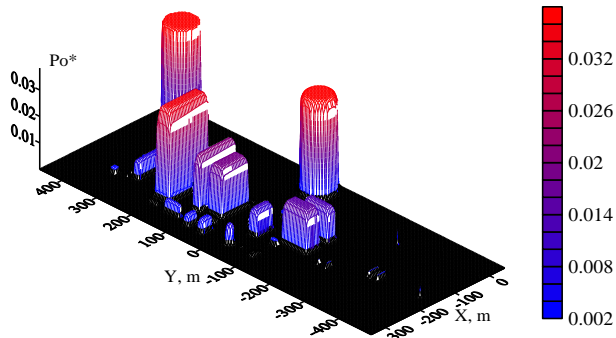
**Figure 7:** Distribution of the coefficient  $P^*$  at  $U_L=-100 \text{ MV}$  ( $P_{\Delta I}=0.02$ )



**Figure 8:** Distribution of the coefficient  $P_o^*$  at  $U_L = -100$  MV ( $P_{\Delta I} = 0.02$ )



**Figure 9:** Distribution of the coefficient  $P^*$  with  $U_L$  from  $-8$  MV to  $-100$  MV



**Figure 10:** Distribution of the coefficient  $P_o^*$  with  $U_L$  from  $-8$  MV to  $-100$  MV

In the paper, several variants of lightning protection systems of the most vulnerable and important parts of the facility – cylindrical oil storage tanks with radius 39m and height 34m have been analysed numerically. To make calculated results more representative, they are expressed in terms of the predicted number of years in which lightning may strike ( $K$  in Table 4) for different height, location and number of lightning air terminals. The calculations are performed at  $N_m = 10$ . In order to avoid lightning strikes to the object, it is proposed to use a catenary wire system (see  $N = 10$  in Table 4 and Figure 11). To determine how the height of lightning air terminal may influence the predicted number of lightning strokes to the protected object, it was varied from  $h = 50$  m to  $h = 120$  m (where  $h$  is the full height of an air terminal above the ground

level). The considered lightning protection systems are shown in Table 5 (numbers  $N$  of systems correspond to numbers  $N$  in Table 4).

**Table 4:** Calculated predicted number of years in which a lightning may strike

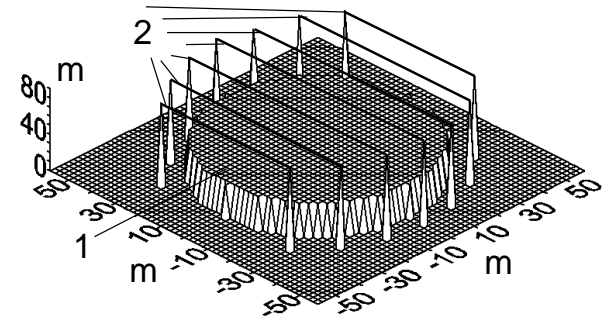
N	Number of lightning air terminals	Type of lightning protection	Height of lightning air terminals, m	K
1	*	-	-	6.3
2	**	LAT	*	6.7
3	1	LAT	60	7.8
4	1	LAT	90	8.1
5	1	LAT	120	8.1
6	2	LAT	90	10.9
7	4	LAT	90	19.3
8	14	LAT	60	42.4
9	14	LAT	90	43.1
10	7	C	50	<100

\* Without lightning protection;

\*\*Existing lightning protection system, which includes 2 lightning air terminals with the heights 91m and 98m (see 3, 4 from Figure 2)  
Type of lightning protection: LAT – Lightning air terminal; C – Catenary wire system.

**Table 5:** Lightning protection systems of the oil storage tank (1- tank; 2 – lightning air terminals)

N=4	N=6
N=7	N=9



**Figure 11:** Lightning protection catenary wire system 2 (1- tank)



## 5 CONCLUSION

In the paper, a numerical prediction technique for the number of lightning strokes to objects protected by lightning air terminals or catenary wire systems has been described. The elaborated mathematical model based on description of the electrical physical process of lightning leader movement to the earth in the last stroke phase, was verified by comparison with the normalised data [1]. The model is used to determine the influence of grounding conditions of a lightning air terminal on its protective radius. The analysis has shown that for the cases when an object is located within the normalized protected zone of an air terminal, its grounding resistance does not influence the probability of lightning attachment up to values  $R_{L1} < 10^4 \Omega$ . However, if the top of an object exceeds slightly the height of the normalized protected zone, this influence may start from  $R_{L1} > 100 \Omega$  for the worst case scenario of lightning spark development.

The proposed model to implement the calculation of lightning stroke probabilities distribution in the territory of an extended oil storage facility has been performed. It has shown the insecurity of the existing lightning protection system as the most vulnerable and important parts of the facility – cylindrical oil storage tanks are supposedly the most hit. The variants of reliable lightning protection means of vulnerable massive oil storage tanks have been analysed. As observed from Table 4, increasing the height of a lightning air terminal from 90m to 120m ( $N=4,5$ , Table 4) does not influence the probability of lightning strokes. However, increasing the number of lightning air terminals from 1 to 14 ( $N=4, 6, 7, 9$ , Table 4) may cause substantial reduction of lightning attachments. The best result is obtained with the application of a catenary wire system ( $N=10$ , Table 4).

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