STEPWISE TRANSIENT RESPONSE OF WINDING STRAY CAPACITANCE

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Abstract: Step response of coil shows that the spike-like current appears at the fast stage of the current in addition to the current which increases slowly depending on the resistance and the inductance. The spike-like current is thought to be a charging current that flows to the capacitor between windings, so-called stray capacitor. In this paper, we report the spike-like current, which appears to the step response of the coil, does not decrease monotonously but stepwise. We have observed a stepwise current not only in the experiment but also the electromagnetic field simulation by Method of Moment. The stepwise decay current is a capacitive current until inductive current appears, as same as the charge current of the capacitor which is formed by the stray capacitance between adjoined turns. However, the stepwise current decay can be also explained by magnetic induction from windings in the neighbourhood of a turn.

1 INTRODUCTION

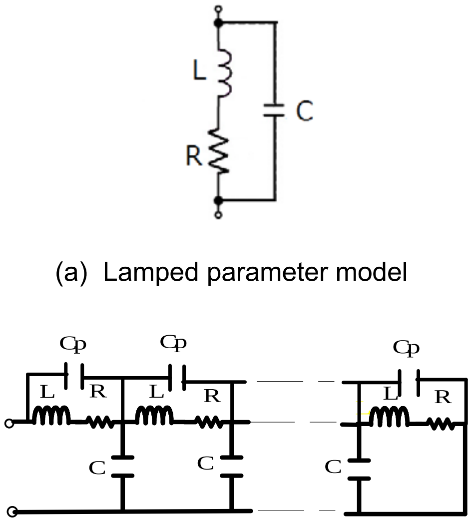
Transformer is one of the popular devices of electrical and electronic circuits, and they have various electrical and geometrical sizes such as from ones in micro machine to ones in 1000kV power transmission lines. The fundamental structure consists of winding and magnetic material to make magnetic field in space, and equipment which is applied the Faraday’s electromagnetic induction law in electromagnetic theory. Energy conversion efficiency and reliability become the important issues as transformers are used for long-term in electric power. The reliability would be ensured according to high withstanding voltage winding arrangements including insulating materials and overvoltage protective devices such as arresters. The electromagnetic characteristic must be cleared to manufacture a compact and highly reliable transformer, and also considered in the circuit in respect of the interaction with other circuit elements. For inverter transformer and resonant voltage transformation circuit like Tesla coil, EMC filters etc. stray capacitance of the coil becomes a problem for the high frequency operation to miniaturize transformers [1-4]. Overvoltage in the power system often originates from the transient or surge, and the analysis including the transformer wiring is required for the overvoltage analysis. Until now, the voltage has been required from the circuit calculation by the expression of the transformer by the equivalent circuit. We applied electromagnetic field analysis method which calculate simultaneously also electric field distribution and magnetic field distribution necessary for the insulation in the transformer to the surge analysis of the coil [5]. According to the result, in addition to the current characteristic as an inductance, the current of the spiking flows at the early stage, when step voltage is applied to the coil. It was regarded as a charging current which flows electrostatic capacity known as stray capacitance of the winding. In this paper, we show that the this spiking current appeared s in step response of the coil is the attenuation current which changes stepwise, and it is tried to explain the results by electromagnetic induction between coils except by electrostatic capacity.

2 EQUIVALENT CIRCUIT OF COIL AND STEP RESPONSE

2.1 Equivalent circuit of coil

It is familiar to present an equivalent circuit of a coil such as the circuit which consists of an electrostatic capacitance, a resistance and a inductance shown in Figure 1. The inductance and the resistance are straightforward represented the original functions as inductor and the resistance of the conducting wire. The electrostatic capacity is stray capacitance [7] which shows the resonance of the coil in high frequency. The following are included for this capacity: electrostatic capacity between terminals and electrostatic capacity between coils. The former is the electrostatic capacity of two metal piece applied voltages in the electrostatics. In the meantime, the electrostatic capacity between coils is hard to be understood because the coil is a conductor, and formed a short-circuit in the direct current level. It is explained that the electrostatic capacity appears, since the potential is different on winding conductors if the wavelength shortens in high frequency. And, the electrostatic capacity is used, because the negative component at the impedance of the spiral conductor appears [4]. The electrostatic capacity between coils is shown by the parallel addition in ladder type equivalent circuit shown in Figure 1(b) which is connected to equivalent circuit of each turn represented by L and R element.

Distributed constant circuit is used as an equivalent circuit, when the propagation of the line is considered, and the electrostatic capacity between winding is shown using Cp as shown in Figure 1(b) in case of the coil. That is to say, it is an equivalent circuit of transmission line which consists of resistance and impedance per unit length which paralleled electrostatic capacity at the inductance and capacity between conductor and ground.



(a) Lamped parameter model

(b) Distributed parameter model

Figure 1: Equivalent circuit model of coil

2.2 Step response

Figure 2 shows typical step current response of coil when it is connected to a step voltage source with voltage E and power supply resistance Rs. The initial pulse is the current which mainly flows at the electrostatic capacity, and the part which approaches asymptotically to the steady-state value E/Rs which flows in the inductance. Generally, the equivalent circuit of Figure of 1 is made from the response like Figures 2. We also examined the stray capacitance of the transformer model by Method of Moment, numerical electromagnetic field analysis [5]. The continuous decay current of the peak is seemed to be the charging current of stray capacitance and appeared also in the computer simulation [5]. In addition, we tried to examine the build-up part in detail by Finite Difference Time Domain Method, and find a stepwise current decay

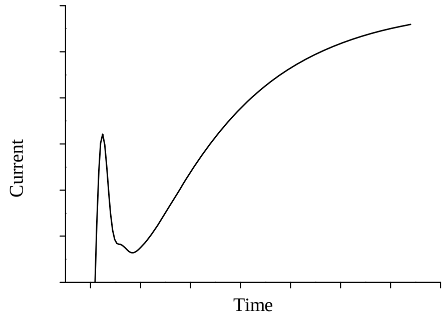


Figure 2: Typical step current response of coil

3 SIMULATED STEP RESPONSE OF MODEL COIL

3.1 Simulation Model

Typical coil is several centimetre in diameter and the conductor is longer than several meters. To measure the transient or surge in typical winding, we may use oscilloscope of the frequency band of several GHz and step voltage source in the rise time of the picosecond. It is difficult to utilize the measurement system which can study such fast process generally.

We decided the geometrical size of the coil to have enough time resolution to simulate considering the resolution time of the step voltage generator and measuring devices used in experiment. The coil is composed of 0.9m in diameter, 0.05m in coil length, and the conductor of 2mm in the diameter. We assumed that the conductor is perfect one and without dielectric cover in simulation.

Method of moment (MoM) was selected from among a lot of numeric electromagnetic field analysis methods. Though the FDTD method is suitable for the analysis of the coil with the core, it is necessary to divide the spatial domain into quite a lot of small cells, and as a result, a large computing time is required. However, method of moment simulates only the conductor by many segments, but smaller linear equation than other method, and hence the computing time becomes small. Another merit of MoM is easy to make the simulation model in 3D space.

The simulated coil model is shown in Figure 3. The Figure is a 3D view of the model in the electromagnetic simulation program FEKO [11].We used about 1500 wire segments to simulate the winding. The conductor is simulated by wire segment approximation of a perfect conductor of 20mm in length and 1mm in the radius.

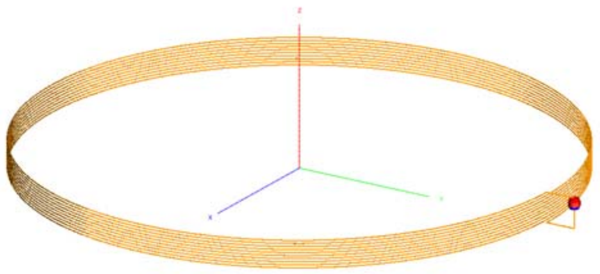


Figure 3: Simulated coil model in Method of Moment

In MoM, electromagnetic field is calculated in frequency domain. Therefore, reverse Fourier transformation is needed to simulate the step response of the current of the conductor. To simulate the time response, Fourier analysis on frequency step 2MHz and 512 points of the number of samples is adopted.

The power supply is assumed the ideal step voltage source of the internal resistance 500Ω. Neither a cable from the pulse voltage generator in experiment, a connector nor a geometrical size of resistance etc. is simulated because we consider that their roles are small to the electromagnetic phenomena in winding, and to avoid too complex model.

3.2 Simulation Result

Figure 4 shows the current response to the step voltage obtained in simulation. The current is approach asymptotically the stationary current which is decided in the ratio of the internal resistance of the source to the step voltage. The pulse current like spike flows at the early stage, and the current of the damped oscillation superimposes the asymptotic current which increases gradually by inductance.

An initial part is expanded in time, and shown in Figure 5. The current decreases along with the elapse of time, which is same process with the charging capacitor. However, it is difficult to think that charging current changes like step because the turns in coil has uniform geometrical and electrical relations between the windings.

The step time found in Figure 5 is about 9ns, and this value is almost the same 9.3 ns in time to propagate one turn in speed of light.

We observed the same step current decrease in simulations which are done in FDTD model with iron-core.

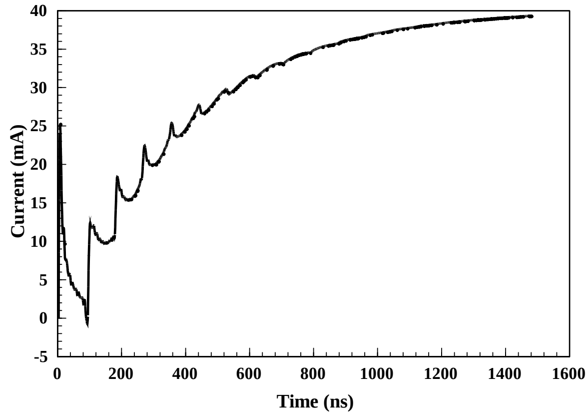


Figure 4: Step response of the air core coil in simulation by Method of Moment

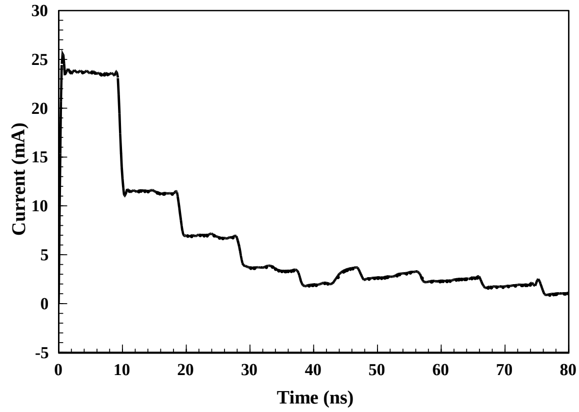


Figure 5: Fast stage of step response of the air core coil in simulation by Method of Moment

4 MEASUEMENT OF STEP RESPONSE OF MODEL COIL

The step response of the model coil was measured experimentally in order to verify whether measurement of step response and attenuation current of the simulation stepped appear.

4.1 Experimental Model

Since in the observation using the low-speed system, the change of the current step becomes gentle, and almost continuously decreases. Therefore, the report of measurement result like Figure 5 is little. High-speed instruments which can be utilized in experiment present in laboratory are the pulse generator of 250ps rise time and a oscilloscope of frequency band with 400MHz.

The experimental coil model is shown in Figure 6. The geometrical configuration and size is the same as the numerical simulation model. The coil was made in single-silk-covered wire of the 2mm diameter.

For the increase of the current propagation time of the 1 turn concerning the above-mentioned time resolution, the ratio of coil length to diameter is small. An output of step voltage source of 1ns or less rise time was connected with the terminal of the coil in the coaxial cable of characteristic impedance 50 Ω. And, to control the decay time of current, the resistance of 512 Ω was inserted between the coil and voltage cables at the output terminal of the cable in the series in respect of matching resistance of 50 Ω.

Figure 6: Air-core coil model in experiment

4.2 Experimental Result

Figure 7 shows a step response of the solenoid. It is a current step response which can be explained by the equivalent circuit of Figure 1. The current peak appears by the parallel electrostatic capacitor at the early stage, and it attenuates in the time constant which depends on the product of an electrostatic capacitance and a power supply resistance. The steady current which depends on the terminal voltage and the circuit series resistance, which is composed of a resistance of power supply and a serial insertion resistance, approaches gradually to the steady state current with the progress in the time.

Figure 8 shows the expanded early stage current. It is clear that the current is almost constant just after having added step voltage and decrease stepwise after about 11ns. It decreases while changing like the step as well as the simulation. The step time is 11ns in time to propagate one turn in speed of light, and it is larger than 9.3ns of simulations. Moreover, the decrease of the second step is smaller though the first step is the same as the simulation.

There are these differences between the simulation and the experiments. However, close agreement between simulated and experimental step current response of the coil was obtained. If the cable and the connector to apply pulse voltage, the insulator, and the skin effect, etc. are not considered, we can a good agreement in the experiment and the simulation, considering simple modelling the use of the wire segment approximation of the perfect conductor in MoM.

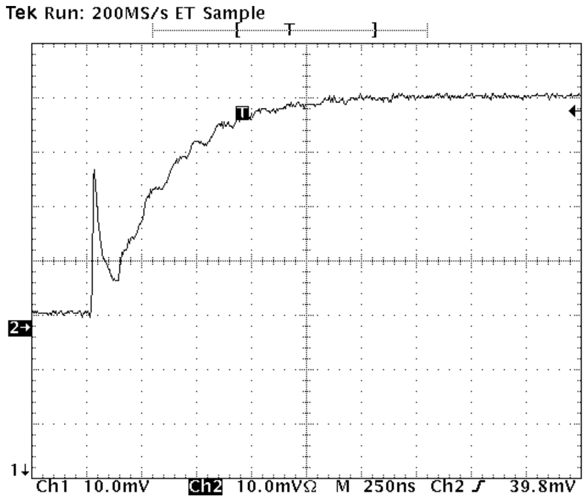


Figure 7: Step response of the air core coil in experiment

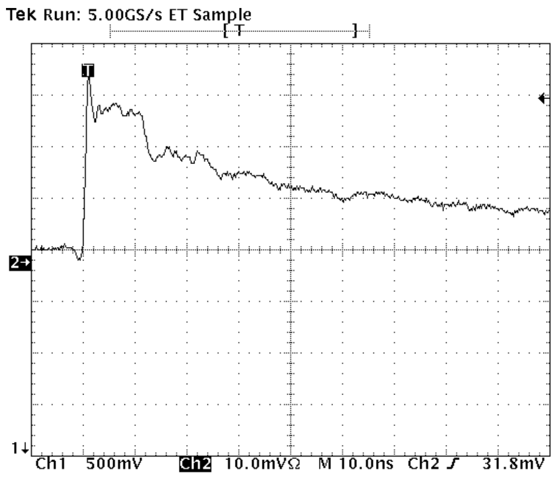


Figure 8: Fast stage of step response of the air core coil in experiment

5 DISCUSSION

Step current from the voltage source flows into the terminals of the coil. The current travels the first turn conductor. As the distance between two adjacent turns is uniform, the current wave may have the constant traveling parameters such as the characteristic impedance and the velocity.

The travelling current which flows for each turn are shown by the following equation, if the current peak value does not change on the way.

$$i_k(t) = I_0 u_1(t - k\tau) \qquad (1)$$

Where : $u_1(t)$ = a unit step function

τ = Travelling time of current to round a turn

I_0 = Peak value of current

We disregard the axial propagation time.

$$\begin{aligned}\phi_{k\ell}(t) &= p_{k\ell}\cdot i_k(t)\cdot v\cdot(t-\tau)\\ &= p_{k\ell}\cdot I_0\cdot u_1(t-k\tau)\cdot v\cdot(t-\tau)\end{aligned} \quad (2)$$

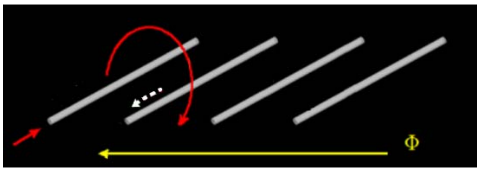
Where: p_{ki} = is the mutual electromagnetic induction coefficient between the i-th turn and n-th turn.

This magnetic flux induces the following back electromotive force by electromagnetic induction.

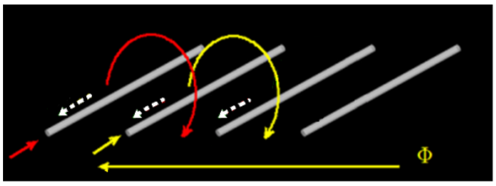
$$e_{kl} = -\frac{d\phi_{kl}}{dt} = -p_{kl}I_0 v u_1(t-k\tau) \quad (3)$$

This voltage produces a reverse current which superposes to the input current in the first turn. Therefore, the input current changes stepwise.

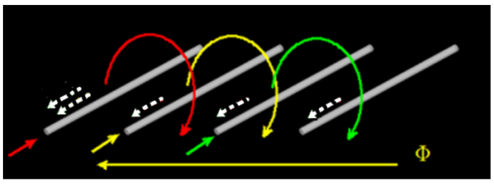
Figure 9 shows the turn current and the magnetic flux represented by a line and an arch where arrow attached. During 0<t< τ , the current keeps the step current I_0 , and next term τ <t<2 τ , the reverse current component induced from 2nd turn current appears, and so on.



(a) 0<t< τ



(b) τ <t<2 τ



(c) 2 τ <t<3 τ

Figure 9: Magnetic induction in each time step

Breit et al [8] studied the equivalent circuit of the coil that mutual induction between winding of the coil is considered. However, it cannot explain the change in the current as shown in Figure 5 by the equivalent circuit of the coil.

It is known that the capacitance between two parallel conductors [7] becomes larger than experimental value, and an experimental equation is proposed [9]. Massarini et al studied equations to calculate self-capacitance between many turns. These capacitance are for the equivalent capacitance shown in Figure.1, and is possible to explain the stepwise current decrease.

6 CONCLUSION

The electric capacitance used for the equivalent circuit of the coil was examined from respect of transients by the electromagnetic field simulation and the experimental measurement. The transient current response shows As a result, when the step voltage was impressed to the coil, it was clarified that the step response current shows the time change of the capacitive current that decreases stepwise. The stepwise current decay can be also explained by magnetic induction from windings in the neighbourhood of a turn.

7 REFERENCES

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