VERIFICATION OF ANALYTICAL EXPRESSIONS FOR CALCULATION OF THE RESISTANCE TO GROUND OF SINGLE CONDUCTIVE CONCRETE ENCASED GROUNDING ELECTRODES BY FEM

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Abstract: Grounding grids are essential for the normal and safe operation of all electrical power systems and networks. For safety reasons, grounding resistance is often estimated by means of analytical or numerical methods via potential distribution calculations along the soil structure. During the last two years of research, mathematical expressions for calculating the resistance to ground of conductive concrete encased vertical and horizontal grounding electrodes have been deduced assuming that the soil structure is isotropic and uniform. All derived mathematical expressions, are compared and validated with results obtained by FEM modeling, realized using the commercial software product ANSYS. Since the Numeric modeling for all types of single conductive concrete encased electrodes is performed without any simplifications, the obtained FEM resistance results have been used as relevant and accurate basis for reliable comparative analysis. It has been concluded in general that the presented inhere mathematical expressions have the necessary accuracy to be implemented in software products and methods for design of nonconventional grounding installations involving conductive concrete encased electrodes.

1 INTRODUCTION

The low-resistivity-materials (LRMs) are widely used to decrease the power frequency grounding resistance of grounding systems in the regions with high soil resistivity. There is experimental data that LRMs can decrease also impulse grounding resistance with 25 - 45% [1, 2]. During the last years of research current-conductive concretes, based on cement, fine sand and black foundry graphite, are developed having the above advantages. It had been verified also that zinc-plated steel electrodes encased in conductive concrete has increased corrosion protection when installed in chemically aggressive soils or regions with increased level of DC stray currents.

Even in the international standards dealing with grounding installations there are no available and reliable analytical expressions and methods for design of grounding installations using electrodes encased in LRMs [3, 4]. This fact up to now makes impossible the large-scale application of such grounding installations.

Formulae for grounding resistance calculation of conductive concrete encased vertical and horizontal grounding electrodes, suitable for grounding design have been deduced assuming that the soil structure is isotropic and uniform [7, 8].

All derived expressions presented inhere are compared and validated with results calculated for the same geometries by FEM modeling, using the commercial software product ANSYS®.

The verified in the present paper mathematical expressions combined with the technical advantages of this relatively new type of grounding electrodes are a prerequisite for the fast application in practice of grounding installations using resistance enhancing current-conductive materials.

2 MATHEMATICAL EXPRESSIONS INTRODUCTION AND THEIR VERIFICATION

2.1 Grounding electrodes description and their mathematical expressions representation.

In the years up to now the following research activities have been followed consequently regarding grounding electrodes encased in conductive concrete: 1) development and investigations of LRMs; 2) physical modeling of conductive concrete encased electrodes and monthly measurements of their resistance [7]; 3) deducing of accurate analytical expressions for calculation the resistance to ground of standard single LRM encased electrodes in uniform soil – according to [8, 9];

The mathematical modeling characterizes the following cases of single electrodes and their resistance:

- *For conventional vertical grounding electrode:*

$$R_{V1} = \frac{\rho_S .tg\alpha}{\pi .m} \ln \frac{d+2m}{d},\ \Omega \tag{1}$$

where $m = 2 \cdot l \cdot tg\alpha$

- *For a vertical grounding electrode encased in one layer of conductive concrete*

$$R_{V2} = \frac{tg\alpha}{\pi .m}\left[\rho_C \ln \frac{(d+2m)D_C}{d(D_C+2m)} + \rho_S \ln \frac{D_C+2m}{D_C}\right],\ \Omega \tag{2}$$

$$R_{V3}=\frac{\rho_C}{2\pi l}\left\{\ln\frac{4l}{d}\left(\sqrt{1+\left(\frac{d}{4l}\right)^2}\right)+\frac{d}{4l}-\sqrt{1+\left(\frac{d}{4l}\right)^2}\right\}+ \frac{\rho_S-\rho_C}{2\pi l}\left\{\ln\frac{4l}{D_C}\left(\sqrt{1+\left(\frac{D_C}{4l}\right)^2}\right)+\frac{D_C}{4l}-\sqrt{1+\left(\frac{D_C}{4l}\right)^2}\right\} \quad (3)$$

- *For a vertical grounding electrode encased in two different concentric layers of conductive concrete*

$$R_{V4}=\frac{tg\alpha}{\pi.m}\left[\begin{array}{l}\rho_{C2}\ln\dfrac{(D_{C1}+2m)D_{C2}}{D_{C1}(D_{C2}+2m)}+\\ +\rho_{C1}\ln\dfrac{(d+2m)D_{C1}}{d(D_{C1}+2m)}+\rho_S\ln\dfrac{D_{C2}+2m}{D_{C2}}\end{array}\right] \quad (4)$$

$$\begin{aligned}R_{V5}=&\frac{\rho_{C1}}{2\pi l}\left\{\ln\frac{4l}{d}\left(\sqrt{1+\left(\frac{d}{4l}\right)^2}\right)+\frac{d}{4l}-\sqrt{1+\left(\frac{d}{4l}\right)^2}\right\}+\\ &+\frac{\rho_{C2}-\rho_{C1}}{2\pi l}\left\{\ln\frac{4l}{D_{C1}}\left(\sqrt{1+\left(\frac{D_{C1}}{4l}\right)^2}\right)+\frac{D_{C1}}{4l}-\sqrt{1+\left(\frac{D_{C1}}{4l}\right)^2}\right\}\\ &+\frac{\rho_S-\rho_{C2}}{2\pi l}\left\{\ln\frac{4l}{D_{C2}}\left(\sqrt{1+\left(\frac{D_{C2}}{4l}\right)^2}\right)+\frac{D_{C2}}{4l}-\sqrt{1+\left(\frac{D_{C2}}{4l}\right)^2}\right\}\end{aligned} \quad (5)$$

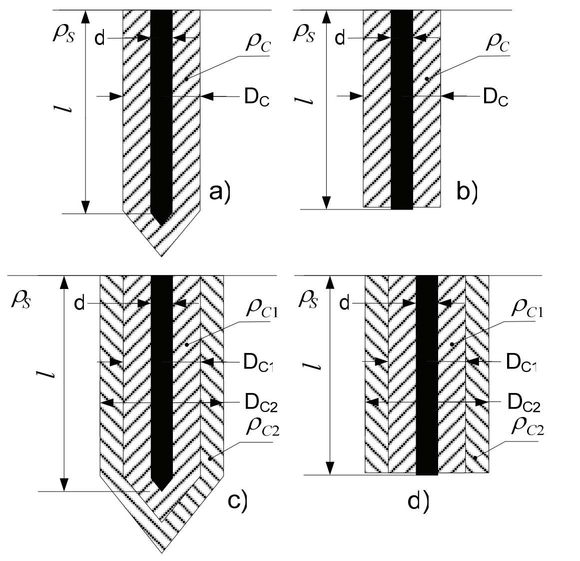

Figure 1: Graphical interpretation of vertical grounding electrodes, modeled mathematically: a)one layer encased electrode according to (2) ; b) one layer encased electrode according to (3); c) two concentric conductive concrete layers encased electrode according to (4); d) two concentric conductive concrete layers encased electrode according to (5).

- *For conductive concrete encased horizontal grounding electrode*

$$R_{H1}=\frac{1}{2\cdot\pi\cdot l_X}\left(\begin{array}{l}\rho_S\ln\dfrac{\pi l^2}{2(x+y)t}+\rho_C\cdot\ln\dfrac{2l^2}{b\cdot t}-\\ -\rho_C\ln\dfrac{\pi l^2}{2(x+y)t}\end{array}\right),\ \Omega \quad (6)$$

$$R_{H2}=\frac{1}{2\cdot\pi\cdot l_X}\left(\rho_S\ln\frac{\pi l^2}{2(x+y)t}+\rho_C\cdot\ln\frac{(x+y)}{b}\right),\ \Omega \quad (7)$$

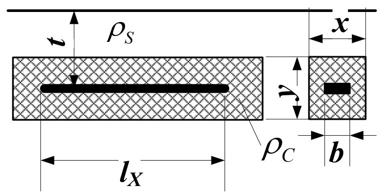

Figure 2: Graphical interpretation of mathematically modeled conductive concrete encased horizontal grounding electrode according to (6) and (7)

The above deduced mathematical expressions are to be verified and compared with results obtained form FEM modeling and other calculated via existing math. expressions used in the world's practice [3, 4], as specified below:

- *For conventional vertical grounding electrode*

$$R_{V6}=\frac{\rho_S}{2\cdot\pi\cdot l}\ln\frac{4l}{d},\ \Omega \quad (8)$$

$$R_{V7}=\frac{\rho_S}{2\pi l}\left\{\ln\frac{4l}{d}\left(\sqrt{1+\left(\frac{d}{4l}\right)^2}\right)+\frac{d}{4l}-\sqrt{1+\left(\frac{d}{4l}\right)^2}\right\} \quad (9)$$

- *For a vertical grounding electrode encased in one layer of conductive concrete*

$$R_{V8}=\frac{1}{2\pi l}\left\{\rho_C\ln\frac{D_C}{d}+\rho_S\left[\ln\left(\frac{8l}{D_C}\right)-1\right]\right\},\ \Omega \quad (10)$$

- *For conventional horizontal bar type grounding electrode*

$$R_{H3}=\frac{\rho_S}{2\cdot\pi\cdot l_H}\cdot\ln\frac{2l^2}{b\cdot t},\ \Omega \quad (11)$$

The used symbols above have the following meanings and values:

$\rho_C = 25\ \Omega.\mathrm{m}$ – resistivity of conductive concrete for grounding electrodes covered with a single conductive concrete encasement;

ρ_S – soil resistivity, Ω.m;

$l = 0,8\ \mathrm{m}$ – length of the vertical grounding electrodes;

$d = 0,027\ \mathrm{m}$ – vertical grounding electrodes diameter;

$D_C = 0,14\ \mathrm{m}$ – external diameter of the conductive concrete encasement for vertical grounding electrode covered with one single layer of conductive concrete;

$\alpha = 30^\circ$ – half of the angle at the vertical electrode top end;

$\rho_{C1} = 23\ \Omega.\mathrm{m}$ – resistivity of the internal conductive concrete layer for vertical grounding electrodes encased in two different concentric conductive concrete layers;

$\rho_{C2} = 13\ \Omega.\mathrm{m}$ – resistivity of the external conductive concrete layer for vertical grounding electrodes encased in two different concentric conductive concrete layers;

$D_{C1} = 0,07\ \text{m} -$ external diameter of the first conductive concrete encasement contacting the metal surface for a vertical grounding electrode covered with two different concentric layers of conductive concrete;

$D_{C2} = 0,14\ \text{m} -$ external diameter of the second conductive concrete encasement contacting the surrounding soil for a vertical grounding electrode covered with two different concentric conductive concrete layers;

$L_H = 2\ \text{m} -$ horizontal bar type grounding electrode length;

$x = 0,1\ \text{m} -$ conductive concrete encasement width (width of the ditch, where the horizontal bar type grounding electrode is mounted);

$y = 0,12\ \text{m} -$ thickness of the conductive concrete encasement for horizontal bar type grounding electrodes;

$t = 0,6\ \text{m} -$ horizontal electrode mounting depth under the soil surface.

2.2 Grounding resistance mathematical expressions verification by FEM simulations

Since the present publication is a natural continuation of the foregoing [6], detailed description of the ANSYS models of conductive-concrete encased electrodes simulated with ANSYS could be found in the full-page version of the ISH 2011 Proceedings.

All FEM results for the resistance to ground are used as reference. The simulations are performed for electrodes with fixed geometrical dimensions assuming that soil structure is isotropic and uniform with a constant resistivity of 52,6 Ω.m.

All resistance values calculated using the analytical expressions presented in 2.1 for the respective geometries are compared with the FEM results and conclusions are made regarding the accuracy of the deduced grounding resistance formulae and the existing widely used ones. The relative error in percents is calculated for each mathematical expression. For better clarity the results are presented in tables.

Table 1: Vertical conventional grounding electrode

Ansys results	Deduced math. expressions		Mathematical expressions used in practice			
	(1)		(8)		(9)	
R_V, Ω	R_{V1}, Ω	ε, %	R_{V6}, Ω	ε, %	R_{V7}, Ω	ε, %
45,09	44,37	-1,59	49,97	10,82	39,59	-12,2

Table 2: Vertical grounding electrode, encased in one layer of conductive concrete

Ansys results	Deduced math. expressions				Math. express-ions used in practice	
	(2)		(3)		(10)	
R_V, Ω	R_{V2}, Ω	ε, %	R_{V3}, Ω	ε, %	R_{V8}, Ω	ε, %
35,99	35,66	-0,93	30,75	-14,6	37,72	4,81

Table 3: Vertical grounding electrode, encased in two concentric layers of conductive concrete

Ansys results	Deduced math. expressions			
	(4)		(5)	
R_V, Ω	R_{V4}, Ω	ε, %	R_{V5}, Ω	ε, %
33,67	33,53	-0,41	28,59	-15,10

Table 4: Horizontal conventional bar type grounding electrode

Ansys results	Mathematical expression used in practice	
	(11)	
R_H, Ω	R_{H3}, Ω	ε, %
22,46	24,32	8,26

Table 5: Horizontal bar type grounding electrode, encased in conductive concrete

Ansys results	Deduced math. expressions			
	(6)		(7)	
R_H, Ω	R_{H1}, Ω	ε, %	R_{H2}, Ω	ε, %
18,79	20,94	11,44	19,56	4,1

The results above are presented graphically in Fig.3 for the vertical electrodes and in Fig. 4 for the horizontal ones.

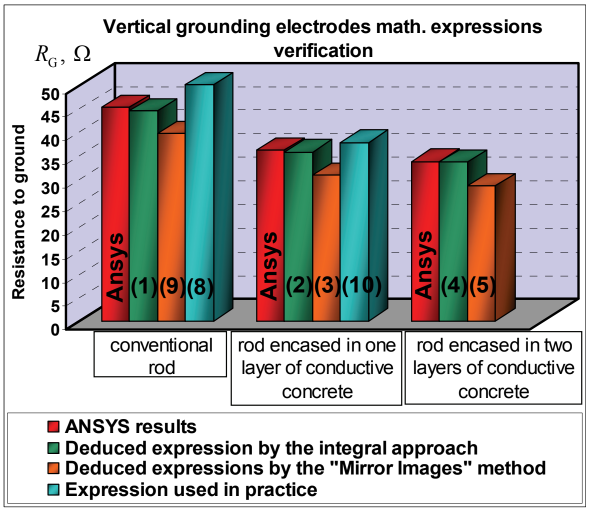

Figure 3: Verification of the mathematical expressions for calculation of the resistance to ground for vertical grounding electrodes

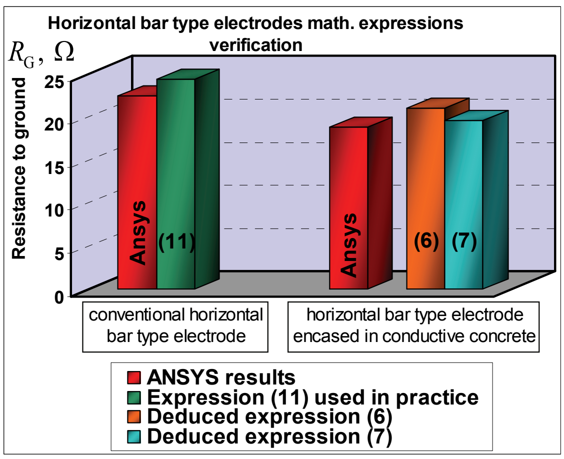

Figure 4: Verification of the mathematical expressions for calculation of the resistance to

ground for horizontal bar type grounding electrodes;

2.2.1. Results discussion for the vertical electrodes

- ✓ The expressions with numbers (1), (2) and (3) for calculation the resistance to ground of vertical conventional, concrete encased and dually concrete encased grounding electrodes, derived by the Integral approach has an relative error not exceeding 1,6 %. These formulae could be used successfully for grounding design;
- ✓ Expression number (8) is widely used in the practice for calculating the resistance of conventional vertical gr. electrodes. Even it is quoted in the international standards IEEE Std 80-2000 and IEEE Std 81-1983, its relative error compared with the ANSYS results is 10,82 %. From one side it could result to grounding installation overdimensioning which means increased metal demand and overall grounding installation costs. From other side, these 10 % are guarantee that the resistance of the grounding system will not exceed the norm regulated value during the dry seasons of the year;
- ✓ Equations (9), (3) and (5) are derived using the method of the “Mirror Images”. Their relative error is between 12 and 15,1 % and is negative. Their use for design purpose could result in serious grounding installation design underdimensioning. That means that the respective grounding installation resistanse to ground will violate in each case the norm regulated resistance value. That’s why the use of these formulae is not recommended;
- ✓ Expression (10), which is quoted in the standard IEEE Std 80-2000, has a relative error not exceeding 5 %. It could be used for grounding installation design.

2.2.2. Results discussion for the horizontal electrodes

- ✓ Expression (11), which is used in practice has a relative error of 8,28 %. Its accuracy is suitable for grounding installation design of conventional grounding installations;
- ✓ Expressions (6) and (7) for calculating the resistance of horizontal bar type electrodes encased in conductive concrete are derived using eq. (11) as basis. Their relative errors are 11,46 % for (6) and respectively 4,1 % for (7);
- ✓ Expression (7) has higher accuracy and then it is advisable to be used in practice, giving grounding installation safe margin of 4,1 %.

2.3 Mathematical expressions verification with results from physical field measurement

Monthly measurements for a period of almost 2 years are performed for physical models of grounding electrodes encased in conductive concrete with dimensions, corresponding to the geometries illustrated in Fig. 1 and Fig.2 and according to [7]. All physical models are installed in the territory of the Technical University of Gabrovo and the complete data from grounding resistance and soil resistivity measurments is collected and processed.

FEM has been used to model and make electrical field simulations of the same geometries using the soil resistivity data, measured monthly.

As a result graphs are built for all of the electrodes which are considered. Every graph consists of resistance curves obtained from monthly measurements, FEM simulations and calculations using the mathematical expressions presented in 2.1.

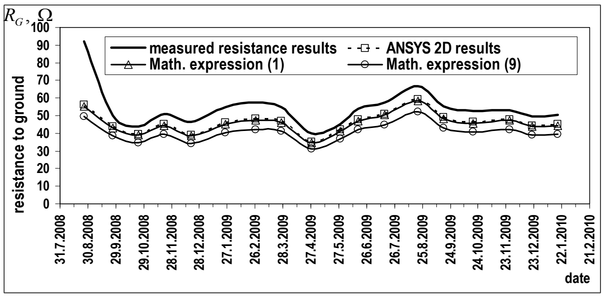

Figure 5: Grounding resistance over time curves for conventional vertical grounding electrode.

From Fig. 5 it could be seen that the Ansys results and these obtained by the implementation of expression (1) matches. Therefore the mathematical and the numeric models give almost the same results. The curve representing expression (9) is below the ANSYS curve, and it is not advisable to be used.

The curve from real physical measurements for that geometry is above the other curves. A reason for that could be the dependence of the upper soil layer resistivity on the seasons and the moisture content.

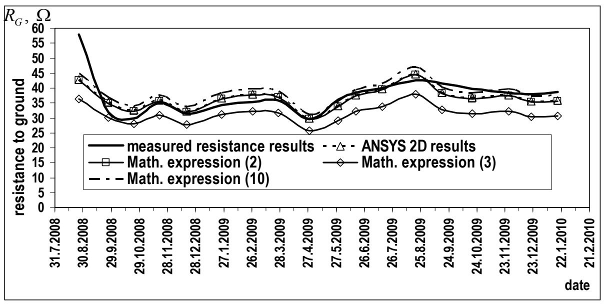

Figure 6: Grounding resistance over time curves for vertical grounding electrode encased in one layer of conductive concrete.

Discussing Fig. 6 it could be noticed that the mathematical model described by expression (2) is the most accurate one. The result of its application matches completely with the ANSYS results and excellently with the real measurements curve. The curve reflecting formula (10) quoted into the international standard IEEE Std. 80 – 2000 is close to the ANSYS results' curve. The average relative error of + 5 % makes expression (10) a good approximation for application in practice in that case;

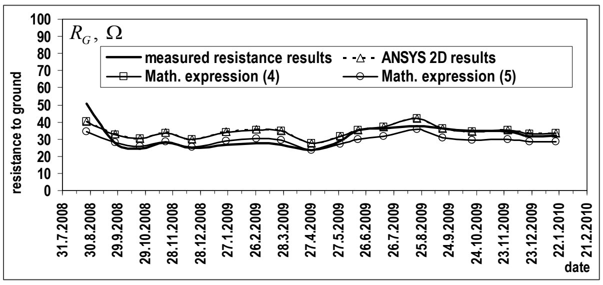

Figure 7: Grounding resistance over time curves for vertical grounding electrode encased in two concentric layers of conductive concrete.

In Fig. 7 for dually encased vertical electrode again the mathematical model described by expression (4) is corresponding exactly to the ANSYS curve. The relative error compared to the real measurements curve is also minimal.

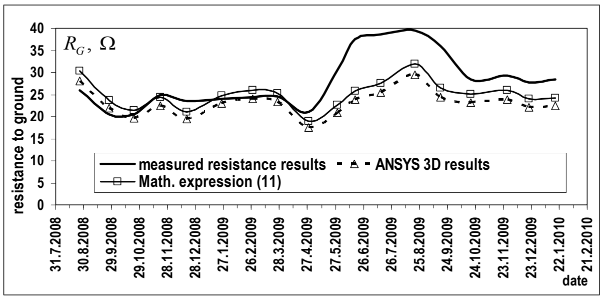

Figure 8: Grounding resistance over time curves for conventional horizontal bar type electrode.

The curve defined implementing the widely used in practice expression (11) has deviation of around 8,26 % related to the ANSYS curve (see Fig. 8). That relative error is completely admissible for design purpose, allowing a safe grounding installation resistance margin. Here it could be noticed that for the period from 27.05. 2009 to 24.10.2009 the measured monthly resistance values are the highest. That deviation is mostly because of the dry summer weather and is typical for conventional horizontal grounding electrodes.

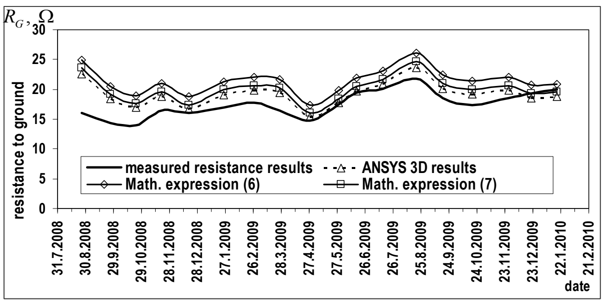

Figure 9: Grounding resistance over time curves for horizontal bar type grounding electrode, encased in conductive concrete.

All modeled resistance curves in Fig. 9 are over the experimental one. The possible reason for that is concrete's hygroscopic behaviour. Normally concrete absorbs water from soil and in that case it decreases its resistivity below the measured and assigned value of 25 Ω.m.The curve representing mathematical expression (7) is closer to the ANSYS one. Its relative error doesn't exceed 5 % and it could be used in practice;

3 CONCLUSIONS

The following conclusions are made taking into consideration the grounding resistance values obtained by real field measurements, these calculated by deduced by the authors math. expressions and results obtained by ANSYS numeric modeling:

- ✓ Math. expressions (1), (2) and (4) for calculating the resistance to ground for conventional vertical electrode, vertical electrode encased in conductive concrete and dually conductive concrete encased vertical electrode, derived using the Integral approach has relative error not exceeding 1,6 %. These expressions are suitable for implementation in grounding design software products.
- ✓ Alternatively, expressions (3), (5) and (9) deduced using the"Mirror Images" method has a negative relative error reaching - 15,1 %. Their use is not advisable, because it could result in serious grounding installation underdimensioning;
- ✓ Expressions (8), (10) and (11) which are published into the international standards [3, 4] normally have a relative error not exceeding + 10 %. They are generally used in practice and have a reasonable safety margin regarding the installations' resistance;

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